Dielectric permittivity of dusty plasma in the Earth's mesosphere

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Key Points:
- An equation for the dielectric permittivity of dusty plasma has been obtained by taking into account the dust charging process and magnetic field.
- The expression is then used to investigate strong radar echoes from the polar summer mesosphere.
- We suggest that the effect of the dust charging process can significantly alter the electromagnetic properties of Earth's mesosphere.

Abstract: This paper deals with the dielectric permittivity of dusty plasma in the earth's mesosphere. We give expressions for the complex dielectric permittivity of dusty plasma, taking into account the effects of the dust charging process and magnetic field. We discuss the dielectric permittivity of dusty plasma in several cases, such as high frequency approximation, parallel propagation in MF/HF band, and effects of plasma movement. Finally, the expressions are employed to study the phenomenon of radar echoes from the polar summer mesosphere. We report that dielectric permittivity caused by the dust charging process gives a radar cross section proportional to ω^{-4} and produces a number density of charged dust that agrees with measurements of mesopheric radar echoes.

Keywords: dusty plasma; dielectric permittivity; polar summer mesosphere

1. Introduction

Polar mesosphere summer echoes (PMSE) are unexpected strong radar echoes from the mesosphere region that have been observed for many decades (Ecklund and Balsley, 1981; Röttger et al., 1988; Hoppe et al., 1988). Since PMSE were first detected in 1979 by the Poker Flat 50 MHz radar, several theories have been put forward to explain them. For a long time it has been speculated that charged dust particles coexisting in large numbers with plasma in the mesosphere are likely to be playing a crucial role in creating PMSE. However, the actual mechanism by which they might do this remains poorly understood. Reviews by Cho et al., 1997 and Rapp et al., 2004 provide detailed information about the characteristics of PMSE and their relation to other mesospheric phenomena.

PMSE have been observed by radar at many frequencies, including the MF (Medium Frequency) band (2–4 MHz), HF (High Frequency) band (3.3–15 MHz), VHF (Very High Frequency) band (49.6–224 MHz), and UHF (Ultra High Frequency) band (500–1290 MHz). Understanding of the existence of PMSE in such a large range of radar frequencies is still a question, particularly when searching for scatters that can produce strong PMSE. Electromagnetic problems arise when charged dust is present in the mesospheric plasma; the influence of dust on the mesospheric dielectric permittivity and radar cross section must be determined. For radar scattering in the VHF/UHF bands, the mesosphere can be treated as an isotropic medium. However, for MF/HF bands, the anisotropic property must be taken into account. Therefore, to study echo mechanisms in such a large band of radar frequency, the effect of the geomagnetic field on the dielectric permittivity of dusty plasma must first be considered. It has been shown that dust charging due to electrons and ions is a highly important physical process after dust is introduced in plasma (Shukla et al., 1996; Duan WS, 2004; Shukla and Mamun, 2015), and the effects of charged dust particles on the dielectric permittivity have been considered by some scholars (Vladimirov, 1994; Shi YX et al., 2007; Jia JS et al., 2015; Li H et al., 2016; Dan L et al., 2017). To our knowledge, however, little effort has been made to investigate the effect of the external magnetic field.

In this paper, we consider dusty plasma in the polar mesosphere to consist of electrons, various species of ions, charged dust, and various neutral species. The following sections will discuss the complex dielectric permittivity, taking the dust charging process into account, in cases involving a geomagnetic field and collision processes. The isotropic dielectric permittivity in VHF/UHF bands and anisotropic dielectric permittivity in MF/HF bands is then explored, followed by a comparison with empirical PMSE measurements.

2. Effect of Dust Charging Process on the Dielectric Permittivity

In this paper, it is assumed that dust particles are spherical and negatively charged; that electrons and dust charge fluctuations

can respond to the electromagnetic perturbation; and that ions cannot respond because of their large mass. The charge of the dust Q is assumed to be governed according to the following charging equation (Shukla and Mamun, 2015):

$$\frac{\partial Q}{\partial t} = I_e + I_i, \tag{1}$$

where the electron and ion charging currents are:

$$
\begin{aligned}
I_e &= I_{e0} + I_{e1} = -e\pi r_d^2 N_e \sqrt{\frac{8kT_e}{\pi m_e}} \exp\left(\frac{e\varphi_0}{kT_e}\right) + I_{e1}(n_{e1}, \varphi_1, \mathbf{E}), \\
I_i &= I_{i0} = -e\pi r_d^2 N_i \sqrt{\frac{8kT_i}{\pi m_i}} \left(1 - \frac{e\varphi_0}{kT_i}\right)
\end{aligned} \tag{2}
$$

and where we assume that the electron current has perturbation $I_{e1}(n_{e1}, \varphi_1, \mathbf{E})$ due to external electric field $\mathbf{E}$, but that the ion dust current is not perturbed, based on the relatively larger mass of dust particles compared to electrons. r_d is the dust grain radius. φ_0 is the equilibrium potential on the surface of the dust grain. $\varphi_0 = CZ_d$, where Z_d and C are the unperturbed dust charge and the capacitance of the dust grain. n_{e1} and φ_1 are the perturbed electron number density and potential. m_e, T_e and N_e are the mass, temperature, and number density of electrons. m_i, T_i and N_i are the mass, temperature, and number density of ions.

The total mean charge Q consists of Z_d and a fluctuating Q_1, governed by the following equation (Shuklaand Mamun, 2015):

$$\frac{\partial Q_1}{\partial t} + v_{ch} Q_1 = I_{e1} = -e \int_{v_m}^{\infty} f_1 v \sigma_d(v, Z_d) d^3 v, \tag{3}$$

where v_m is the minimum velocity at which electrons can reach the surface of dust particles and v_{ch} is the charge relaxation rate of dusty plasma, or the so-called dust charging rate, introduced by Shukla and Mamun, 2015:

$$v_{ch} = e |I_{e0}| \left(\frac{1}{CkT_e} + \frac{1}{CkT_i - eZ_d}\right), \tag{4}$$

where f_1 is the perturbed distribution function given by the solution of the Boltzmann equation. σ_d is the effective collision cross section of the dust with electrons.

$$\sigma_d(v, Z_d) = \pi r_d^2 \left(1 + \frac{v_m^2}{v^2}\right). \tag{5}$$

According to the Equations (1) to (5), neglecting the magnetic field, the dielectric susceptibility χ_d due to dust charging is written as (Shi YX et al., 2007):

$$\chi_d = \frac{\omega_{pe}^2}{j\omega(j\omega + v_e)} \frac{\omega_{ed}}{j\omega + v_{ch}}, \tag{6}$$

where ω_{ed} is given by the following equation (Ma JX and Yu MY, 1994a; Ma JX and Yu MY, 1994b):

$$\omega_{ed} = \frac{8}{3} \sqrt{\pi} r_d^2 N_D V_{Te} \left(1 - \frac{eZ_d}{2T_eC}\right) \exp\left(\frac{eZ_d}{T_eC}\right) \tag{7}$$

and ω_{pe} is the well-known plasma frequency, $\omega_{pe} = N_e e^2/m_e \varepsilon_0$. v_e is the average total collision frequency for electrons with all other species including dust; r_d, N_D, v_{Te} represent the dust radius, dust density, and electron thermal velocity. According to the paramet-

ers of the PMSE region, v_{ch} and ω_{ed} are estimated to be of the same magnitude; v_e is estimated to be about 10^5/s. The dielectric permittivity of electron-ion plasma is given by

$$\chi_p = \frac{\omega_{pe}^2}{j\omega(j\omega + v_e)}. \tag{8}$$

Combined with Equation (6) and (8), the complex dielectric permittivity of dusty plasma without magnetic field can thus be written as

$$\varepsilon = 1 + \chi_p + \chi_d = 1 + \frac{\omega_{pe}^2}{j\omega(j\omega + v_e)} \left(1 + \frac{\omega_{ed}}{j\omega + v_{ch}}\right). \tag{9}$$

The third term (χ_d) on the right-hand side of Equation (9) is the dielectric permittivity, which contains only the dust charging process. Without dust or with a dust charge equal to zero, we have $\omega_{ed} = 0$, and Equation (9) can be reduced to the dielectric permittivity of general electron-ion plasmas.

3. Dielectric Permittivity Tensor of Dusty Plasma Containing Magnetic Field

Taking the magnetic field and simultaneous collision process into consideration, following the work of Yeh and Liu (1972), the complex dielectric permittivity can be written directly as:

$$
\varepsilon = \mathbf{I} - \frac{\dfrac{X}{U}\left(1 - j\dfrac{\xi_d}{U_{ch}}\right)}{1 - Y^2/U^2}
\begin{bmatrix}
1 - Y_x^2/U^2; & -Y_xY_y/U^2 + jY_z/U; & -Y_xY_z/U^2 - jY_y/U \\
-Y_xY_y/U^2 - jY_z/U; & 1 - Y_y^2/U^2; & -Y_yY_z/U^2 + jY_x/U \\
-Y_xY_z/U^2 + jY_y/U; & -Y_yY_z/U^2 - jY_x/U; & 1 - Y_z^2/U^2
\end{bmatrix}
\tag{10}
$$

where $\mathbf{I}$ is the unit tensor, ε is the dielectric permittivity tensor, and

$$U = 1 - jv_e/\omega, \quad U_{ch} = 1 - jv_{ch}/\omega,$$

$$Y = \frac{\omega_{eB}}{\omega}, \quad X = \frac{\omega_{pe}^2}{\omega^2}, \quad \xi_d = \frac{\omega_{ed}}{\omega}, \quad \omega_{eB} = -\frac{e}{m_e}\mathbf{B},$$

where ω_{eB} is the gyro-magnetic frequency.

In the mesosphere, dusty plasma is known to move under the effect of background wind or electric fields, affecting the radar spectral width during PMSE events. Considering a plasma velocity v_p for species i, dielectric permittivity is obtained by replacing ω with $\omega - \mathbf{k} \cdot v_p$ in the above equations. Here, $\mathbf{k}$ is the wave vector.

In the case neglecting collision, $v_e/\omega \ll 1$, from Equation (10) we have:

$$
\varepsilon = \mathbf{I} + \frac{-X + jX\xi_d/U_{ch}}{1 - Y^2}
\begin{bmatrix}
1 - Y_x^2; & -Y_xY_y + jY_z; & -Y_xY_z - jY_y \\
-Y_xY_y - jY_z; & 1 - Y_y^2; & -Y_yY_z + jY_x \\
-Y_xY_z + jY_y; & -Y_yY_z - jY_x; & 1 - Y_z^2
\end{bmatrix}.
\tag{11}
$$

We consider another case, $v_e/\omega \gg 1$, $\omega \approx |\omega_{eB}|$ and propagation parallel to the magnetic field:

$$\varepsilon = I + \frac{-X + jX\xi_d/U_{ch}}{1 - Y^2}\begin{bmatrix} 1; & jY; & 0 \\ -jY; & 1; & 0 \\ 0; & 0; & 1 - Y^2 \end{bmatrix}. \tag{12}$$

The above equation is valid for study of the MF/HF band of the PMSE. Similarly, replacing ω with $\omega - \boldsymbol{k} \cdot \upsilon_p$ in the above equation takes into account plasma movement for studying radar spectral width at MF/HF frequencies.

4. Discussion

At high frequency approximation, $\omega \gg v_e$, and $\omega \gg \omega_{eB}$, and neglecting the effects of the magnetic field, according to Equation (12), the following formula is determined:

$$\varepsilon = 1 - X + jX\xi_d/U_{ch}. \tag{13}$$

Equation (13) is valid for the study of the VHF/UHF PMSE. It is clear that the second term on the right-hand side of the formula leads to a frequency dependence of the real dielectric permittivity proportional to ω^{-2}. Additionally, the real part of the third term leads to a frequency dependence proportional to ω^{-4} and proportional to the product $Z_d N_d r_d^2$.

If considering the radar spectral width due to Doppler shift of plasma, ω must be replaced with $\omega - \boldsymbol{k} \cdot \upsilon_p$ in the above Equation (8).

$$\varepsilon = 1 - \frac{\omega_{pe}^2}{(\omega - \boldsymbol{k} \cdot \upsilon_p)^2} + j\frac{\omega_{pe}^2 \omega_{ed}}{(\omega - \boldsymbol{k} \cdot \upsilon_p)^2(\omega - \boldsymbol{k} \cdot \upsilon_p - jv_{ch})}. \tag{14}$$

Here, the collision frequency is neglected. Equation (14) is thus valid for the study of radar spectral width during PMSE events.

As a comparison with PMSE measurements, the dependence of the PMSE on radar operational frequencies is provided in Figure 1 (Li HL et al., 2009). Based on deduced reflectivity from the PMSE events at several radars (see Table 1 in Rapp and Lübken, 2004), Figure 1 shows the frequency dependence of measured reflectivity, which is theoretically proportional to the radar cross section in
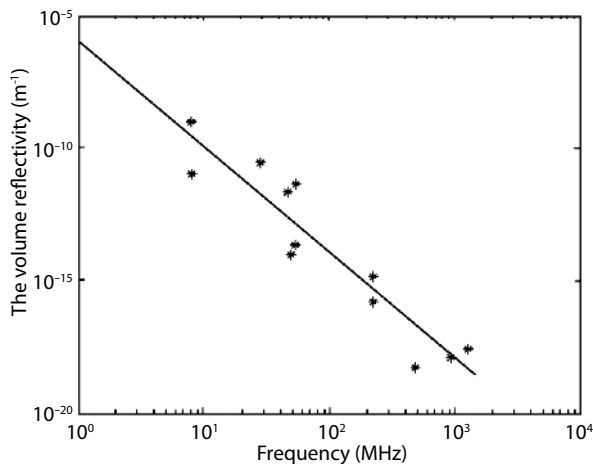
unit volume. The measurements were performed at various times and places, using different radars. It can be clearly observed in Figure 1 that reflectivity for the PMSE region is basically proportional to ω^{-4}, but not to ω^{-7}, as predicted by air turbulence theory in the viscous sub-range. It is known that, if PMSE were due simply to reflection by a layer of dusty plasma with dielectric permittivity governed by the third term in the right-hand side of Equation (14), even without irregularities, the reflectivity would be proportional to ω^{-4}. The effect of irregularity will slightly modify the frequency dependence, as shown in Figure 1. This implies that the PMSE can be produced mainly by a dusty plasma layer in the PMSE region at an altitude that exhibits a very sharp edge, as measured with the ALOMAR SOUSY VHF radar and simultaneous rocket sounding ECT02 (Lübken et al., 1998; Rapp et al., 2003a, b; Rapp and Lübken, 2004; Havnes et al., 1996).

In addition, the PMSE strength in dB is suggested as being proportional to a proxy $P = |Z_d|N_d r_d^2$ based on simultaneous radar and rocket measurements (Rapp et al., 2003b). If the third term in the right-hand side of Equation (14) is considered as the main factor producing the PMSE, it leads to the strength of PMSE being proportional to $(Z_d N_d r_d^2)^2$, which corresponds well with observations.

5. Summary

In this paper, expressions of complex dielectric permittivity for dusty plasma, considering the effects of the dust charging process and magnetic field, are derived and are employed to study the phenomenon of radar echoes from the polar summer mesosphere. We conclude that dielectric permittivity caused by the dust charging process gives a radar cross section proportional to ω^{-4} and is consistent with a number density of charged dust that agrees with measurements of mesopheric radar echoes.

Acknowledgments

Figure 1. Measuring frequency dependence of reflectivity and the result of polynomial fit (Li HL et al., 2009). Symbols stand for the reflectivity and the straight line represents the result of a polynomial fit.

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