

Sensitivity analysis of the lithospheric magnetic field at satellite altitude: the effects of the inducing field and the shape of the magnetic lithosphere

JinSong Du^{1,2,3}, YuKun Li¹, HouPu Li^{4*}, ChangQing Yuan¹, KangAn Zhao¹, JiangSong Gui¹, Pan Zhang¹, and ShaoFeng Bian⁴

¹Hubei Subsurface Multi-scale Imaging Key Laboratory, School of Geophysics and Geomatics, China University of Geosciences, Wuhan 430074, China;

²Key Laboratory of Geological Survey and Evaluation of Ministry of Education, China University of Geosciences, Wuhan 430074, China;

³State Key Laboratory of Geological Processes and Mineral Resources, China University of Geosciences, Wuhan 430074, China;

⁴College of Electrical Engineering, Naval University of Engineering, Wuhan 430033, China

Key Points:

- Alternative formulae for forward calculating the global lithospheric magnetic field in the spherical harmonic domain are proposed.
- The impacts of the inducing field and geometric shape of the magnetic lithosphere on the global lithospheric magnetic field are analyzed.
- The formulae presented will be a potential tool for quantitative interpretation of the Earth's global lithospheric magnetic field.

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Abstract: As a means of quantitative interpretation, forward calculations of the global lithospheric magnetic field in the Spherical Harmonic (SH) domain have been widely used to reveal geophysical, lithological, and geothermal variations in the lithosphere. Traditional approaches either do not consider the non-axial dipolar terms of the inducing field and its radial variation or do so by means of complicated formulae. Moreover, existing methods treat the magnetic lithosphere either as an infinitesimally thin layer or as a radially uniform spherical shell of constant thickness. Here, we present alternative forward formulae that account for an arbitrarily high maximum degree of the inducing field and for a magnetic lithosphere of variable thickness. Our simulations based on these formulae suggest that the satellite magnetic anomaly field is sensitive to the non-axial dipolar terms of the inducing field but not to its radial variation. Therefore, in forward and inverse calculations of satellite magnetic anomaly data, the non-axial dipolar terms of the inducing field should not be ignored. Furthermore, our results show that the satellite magnetic anomaly field is sensitive to variability in the lateral thickness of the magnetized shell. In particular, we show that for a given vertically integrated susceptibility distribution, underestimating the thickness of the magnetic layer overestimates the induced magnetic field. This discovery bridges the greatest part of the alleged gap between the susceptibility values measured from rock samples and the susceptibility values required to match the observed magnetic field signal. We expect the formulae and conclusions of this study to be a valuable tool for the quantitative interpretation of the Earth's global lithospheric magnetic field, through an inverse or forward modelling approach.

Keywords: lithospheric magnetic field; forward calculation; spherical harmonic analysis; sensitivity analysis; satellite magnetism

1. Introduction

Thanks to continuous geomagnetic field measurements provided by satellites (e.g., the Ørsted, CHALLENGING Minisatellite Payload [CHAMP], and *Swarm* constellation of three satellites, as well as the China Seismo-Electromagnetic Satellite-1 [CSES-1] and Macau Science Satellite-1 [MSS-1]), and advances in the methodologies

and techniques of data processing and geomagnetic field modelling, global lithospheric magnetic field models have become progressively more reliable (e.g., Olsen et al., 2017; Sabaka et al., 2020; Thébaud et al., 2021; Jiang Y et al., 2024). These advances make the quantitative interpretation of the lithospheric magnetic field in terms of large-scale magnetic structures in the crust and uppermost mantle (see, e.g., Fox Maule et al., 2005; Thébaud and Vervelidou, 2015; Williams and Gubbins, 2019; Wang J et al., 2024) more feasible than ever before.

Many different approaches have been proposed to infer information about the magnetized sources from magnetic field measurements, such as forward modelling, inverse modelling, statistical

First author: J. S. Du, jinsongdu@cug.edu.cn

Correspondence to: H. P. Li, lihoup1985@126.com

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analysis, correlation analysis, and Euler deconvolution, etc. (see, e.g., Langel and Hinze, 1998, as well as Purucker and Whaler, 2015 for reviews on this topic). Among all the quantitative interpretation methods, considering the high computational efficiency here, we choose to work with the method proposed by Arkani-Hamed and Strangway (1985), which directly converts a global magnetic anomaly map into a magnetic susceptibility contrast map in a Spherical Harmonic (SH) domain. More recently, Gubbins et al. (2011) provided a simple formula that converts a SH magnetic field model into an apparent vector magnetization distribution by means of vector SHs. This approach was later utilized to forward modelling the lithospheric magnetic anomaly field at satellite altitudes (e.g., Masterton et al., 2013; Liu PF et al., 2023). However, the formula derived by Arkani-Hamed and Strangway (1985) additionally allows the direction of the inducing field to be constrained, and thus can be more useful in the inversion of the Earth's global large-scale lithospheric magnetic anomaly field, which is dominated by induced magnetization (Maus and Haak, 2002).

The method by Arkani-Hamed and Strangway (1985) assumes that the Earth's lithosphere is magnetized by a geocentric axial dipole. The corresponding formulae have been utilized by Maus and Haak (2003) as well as Thébaud and Vervelidou (2015) to seek magnetic field annihilators and construct statistical power spectra of the Earth's lithospheric magnetic field, respectively. Nolte and Siebert (1987) and Arkani-Hamed and Dyment (1996) have proposed different formulae that take into account the non-axial dipolar terms of the inducing field. The former method uses only degrees 1–3 of the core field, whereas the latter utilizes all degrees of the core field. However, these formulae are relatively complicated. Moreover, none of the aforementioned methods allow for a magnetic lithosphere of laterally varying thickness.

In this article, we propose alternative forward formulae in terms of SHs that account for an arbitrarily high maximum degree of the inducing field and for a magnetic lithosphere of variable thickness. To be able to separately quantify the effects of the non-dipolar terms of the inducing field and the lateral variations of the magnetic thickness, we derive formulae for all four configurations: (1) a dipolar inducing field and magnetized layer of constant thickness (Section 2.1.1); (2) a dipolar inducing field and magnetized layer of variable thickness (Section 2.1.2); (3) an inducing field of arbitrarily high SH degree and magnetized layer of constant thickness (Section 2.2.1); (4) inducing field of arbitrarily high SH degree and magnetized layer of variable thickness (Section 2.2.2). The formulae in Section 2.1.1 were previously derived by Arkani-Hamed and Strangway (1985), Nolte and Siebert (1987) and Thébaud and Vervelidou (2015). We repeat them here in order to introduce the methodology used to derive the expressions of the more complex configurations of the following sections. The details of the derivation of all formulae are given in the Supplementary Material. In Section 3, we quantify the impact of the geometry of both the inducing field and the magnetic shell on the global lithospheric magnetic field by means of synthetic models. In Section 4, we discuss the reason why the power of the forward modelled lithospheric magnetic field is less than that of the observed field in the literature. And finally, in Section 5, we present some conclusions and suggest potential

applications of the study results.

2. Methods

The magnetic potential V of the lithosphere at point $P(r, \theta, \lambda)$ above the Earth's surface can be represented in SHs (e.g., Backus et al., 1996) by

$$V(r, \theta, \lambda) = r_E \sum_{l,m} \left(\frac{r_E}{r} \right)^{l+1} (g_{\text{Lith},l}^m \cos m\lambda + h_{\text{Lith},l}^m \sin m\lambda) P_l^m(\cos \theta), \quad (1)$$

where $r_E = 6371.2$ km is the geomagnetic reference radius of the Earth; r , θ , and λ are the radius, co-latitude, and longitude, respectively, in the geocentric spherical coordinate system; $g_{\text{Lith},l}^m$ and $h_{\text{Lith},l}^m$ are the Gauss coefficients of degree l and order m of the lithospheric magnetic field, and $P_l^m(\cos \theta)$ is the Schmidt semi-normalized associated Legendre function. The Gauss coefficients of the observed magnetic field produced by a spherical shell of magnetized material with the magnetization vector $\mathbf{M}(r', \theta', \lambda')$, when $r > r'$, can be expressed as (see Equation (7) of Gubbins et al., 2011)

$$[g_{\text{Lith},l}^m, h_{\text{Lith},l}^m] = \frac{\mu_0}{4\pi} \frac{1}{r_E^{l+2}} \oint_{\Omega'} \int_{r_1}^{r_2} \mathbf{M}(r', \theta', \lambda') \cdot \nabla' \left\{ r'^l [\cos m\lambda', \sin m\lambda'] \right\} P_l^m(\cos \theta') r'^2 dr' d\Omega', \quad (2)$$

where r_1 and r_2 are the geocentric radii of the lower and upper surfaces, respectively, of the magnetic spherical shell; $d\Omega' = \sin \theta' d\theta' d\lambda'$; ∇' is the gradient operator applied on $\mathbf{r}'$; and μ_0 is the permeability of free space ($4\pi \times 10^{-7} \text{ N} \cdot \text{A}^{-2}$).

2.1 Considering an Axial Dipolar Inducing Field

In this section, the inducing field is considered as an axial dipolar field. Both its lateral variation and radial variation are taken into account.

2.1.1 A magnetized layer of constant thickness

We first assume that the inducing field is a geographic axial dipole and that the thickness of the magnetized spherical shell is constant and equal to $d = R_2 - R_1$ (see Figure 1a). In this case, the vector of the induced magnetization, when neglecting the effect of the induced field itself, is given by (see, e.g., Equation (5.10) of Blakely 1995, and Equation (4) of Thébaud and Vervelidou 2015)

$$\begin{aligned} \mathbf{M}_{\text{Dipole}}(r', \theta', \lambda') &= \frac{1}{\mu_0} \kappa(r', \theta', \lambda') \mathbf{B}_{\text{Core,Dipole}}(r', \theta', \lambda') \\ &= \frac{1}{\mu_0} \kappa(r', \theta', \lambda') g_{\text{Core},1}^0 \left(\frac{r_E}{r'} \right)^3 (2\cos \theta' \mathbf{e}_r + \sin \theta' \mathbf{e}_\theta), \end{aligned} \quad (3)$$

where $\kappa(r', \theta', \lambda')$ is the scalar magnetic susceptibility distribution, $\mathbf{B}_{\text{Core,Dipole}}$ stands for the dipolar inducing field, $g_{\text{Core},1}^0$ is the first degree and zero order coefficient of SH expansion of the inducing field, $(r_E/r')^3$ describes the radial variation of the strength of the dipolar inducing field, and $\mathbf{e}_r$ and $\mathbf{e}_\theta$ are the radially upward and southward unit vectors on the sphere, respectively. We then decompose the susceptibility variation into its radial and lateral parts as

$$\kappa(r', \theta', \lambda') = f(r') \bar{\kappa}(\theta', \lambda'), \quad (4)$$

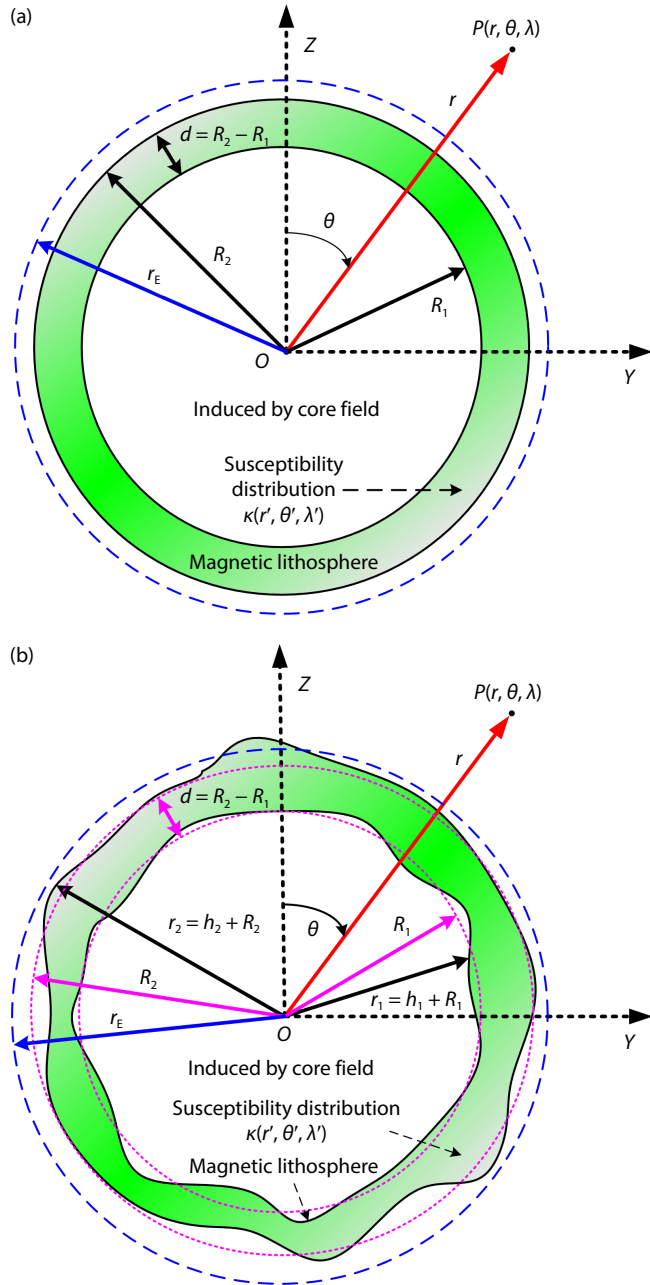


Figure 1. The geometry of a magnetized shell with a spatially varying susceptibility distribution and (a) constant thickness or (b) variable thickness. The magnetized shell, also referred to in the text as the magnetic lithosphere, is denoted by the green layer.

and expand $\tilde{\kappa}(\theta', \lambda')$ in terms of SHs. This gives

$$\kappa(r', \theta', \lambda') = f(r') \sum_{p,q} (\alpha_p^q \cos q\lambda' + \beta_p^q \sin q\lambda') P_p^q(\cos \theta'), \quad (5)$$

where α_p^q and β_p^q are the SH expansion coefficients of degree p and order q . Inserting Equations (3)–(5) into Equation (2) and after some algebraic manipulations, we obtain (see details in Supporting Text A of Supplementary Material)

$$[g_{\text{Lith},l}^m, h_{\text{Lith},l}^m] = g_{\text{Core},1}^m F_l(R_1, R_2) \{ \xi_{l+1}^m [\alpha_{l+1}^m, \beta_{l+1}^m] + \zeta_{l-1}^m [\alpha_{l-1}^m, \beta_{l-1}^m] \}, \quad (6a)$$

where

$$\xi_{l+1}^m = \frac{3l\sqrt{(l+1)^2 - m^2}}{(2l+1)(2l+3)}, \quad \zeta_{l-1}^m = \frac{(l-1)\sqrt{l^2 - m^2}}{(2l+1)(2l-1)}, \quad (6b)$$

$$\text{and } F_l(R_1, R_2) = \frac{1}{r_E^{l-1}} \int_{R_1}^{R_2} f(r') r'^{l-2} dr'.$$

Assuming now $f(r') = 1$, that is, the susceptibility of the magnetized lithosphere does not vary radially, $F_l(R_1, R_2)$ becomes

$$F_l(R_1, R_2) = \left(\frac{R_2}{r_E} \right)^{l-1} \frac{1 - (R_1/R_2)^{l-1}}{l-1}. \quad (6c)$$

The expression above is singular for $l = 1$ but converges to $-\ln(R_1/R_2)$ (see, e.g., Equation (9) of [Thébault and Vervelidou 2015](#)). If $R_2 - R_1 \ll R_1$, $-\ln(R_1/R_2)$ approximately equals zero, which means that for a very thin spherical shell, the induced magnetic field of $l = 1$ is very weak.

Equations (6a)–(6c) were previously derived by [Arkani-Hamed and Strangway \(1985\)](#), [Nolte and Siebert \(1987\)](#), and [Thébault and Vervelidou \(2015\)](#). On the basis of these equations, we derive in the following sections analytical expressions for the SH coefficients of the lithospheric magnetic field for more complex configurations.

2.1.2 A magnetized layer of variable thickness

In this section, we consider the case of a magnetized spherical shell of variable thickness, as shown in [Figure 1b](#). The lower and upper radii of the magnetized shell are expressed by $r_1(\theta', \lambda')$ and $r_2(\theta', \lambda')$, respectively, with

$$r_{[1,2]}(\theta', \lambda') = R_{[1,2]} + h_{[1,2]}(\theta', \lambda'), \quad (7)$$

where $h_1(\theta', \lambda')$ and $h_2(\theta', \lambda')$ are the topography variations of the lower and upper surfaces relative to their average radii (i.e., R_1 and R_2 , respectively; see [Figure 1b](#)). This model can be decomposed into three parts, that is, a spherical shell with a constant thickness d , where $d = R_2 - R_1$, and two magnetic interfaces with topography variations of $h_1(\theta', \lambda')$ and $h_2(\theta', \lambda')$. According to the superposition property of the magnetic potential, we can then write

$$[g_{\text{Lith},l}^m, h_{\text{Lith},l}^m] = [\text{up} g_{\text{Lith},l}^m, \text{up} h_{\text{Lith},l}^m] + [\text{mid} g_{\text{Lith},l}^m, \text{mid} h_{\text{Lith},l}^m] - [\text{low} g_{\text{Lith},l}^m, \text{low} h_{\text{Lith},l}^m], \quad (8)$$

where the subscript expressions up, mid and low denote the upper interface, middle shell and lower interface, respectively. The global magnetic field of the middle shell can be calculated by using Equation (6). Following the same procedure as before, and using the binomial theorem to expand the term $\frac{(1 + h_{[1,2]}/R_{[1,2]})^{l-1} - 1}{l-1}$ (see, e.g., Equation (7) in [Wieczorek and Phillips 1998](#)), we obtain (see details in Supporting Text B of Supplementary Material)

$$[\text{low}, \text{up}] g_{\text{Lith},l}^m, [\text{low}, \text{up}] h_{\text{Lith},l}^m = g_{\text{Core},1}^m \sum_{n=1}^{l-1} F_l(R_1, R_2, n) \left\{ \xi_{l+1}^m [\text{low}, \text{up}] \alpha_{l+1,n}^m, [\text{low}, \text{up}] \beta_{l+1,n}^m \right\} + \zeta_{l-1}^m [\text{low}, \text{up}] \alpha_{l-1,n}^m, [\text{low}, \text{up}] \beta_{l-1,n}^m, \quad (9a)$$

where

$$F_l(R_1, R_2, n) = \left(\frac{R_{[1,2]}}{r_E} \right)^{l-1} \frac{\prod_{j=1}^n l-j}{(l-1)n!}, \quad (9b)$$

with n is the expanding power order of $\frac{(1 + h_{[1,2]}/R_{[1,2]})^{l-1} - 1}{l-1}$ (see Equation (B3)), ξ_{l+1}^m and ζ_{l-1}^m are given by Equation (6b), and $\alpha_{p,n}^q$ and $\beta_{p,n}^q$ are the SH expansion coefficients of

$$\kappa(\theta', \lambda') [h_{[1,2]}(\theta', \lambda')/R_{[1,2]}]^n = \sum_{p,q} \left(\begin{matrix} [\text{low,up}] \alpha_{p,n}^q \cos q\lambda' \\ + [\text{low,up}] \beta_{p,n}^q \sin q\lambda' \end{matrix} \right) P_p^q(\cos \theta'). \quad (9c)$$

In fact, the two interfaces can be expanded up to different power orders. There is a singularity in Equation (9b) when $l = 1$, therefore, in this case,

$$\kappa(\theta', \lambda') \ln [1 + h_{[1,2]}(\theta', \lambda')/R_{[1,2]}] = \sum_{p,q} \left(\begin{matrix} [\text{low,up}] \alpha_{p,n}^q \cos q\lambda' \\ + [\text{low,up}] \beta_{p,n}^q \sin q\lambda' \end{matrix} \right) P_p^q(\cos \theta'), \quad (9d)$$

and Equation (9b) becomes $F_l(R_1, R_2, n) = 1$. According to Equation (9b), the maximum expanding power order n is $l - 1$. For

$$h_{[1,2]}(\theta', \lambda') \ll R_{[1,2]},$$

the forward calculation converges fast (see Equation (B3)). We discuss the impact of the expanding power order on forward calculation in Section 3.3.

2.2 Considering an Inducing Field of Higher Degree and Order

In this section we consider an inducing field of arbitrarily high degree. Two sets of formulae are obtained by considering or not considering the radial variation of the inducing field.

2.2.1 A magnetized layer of constant thickness

In the case of a magnetized layer with constant thickness, it can be shown (see Supporting Text C of Supplementary Material for details) that the Gauss coefficients of the induced lithospheric magnetic field are given by

$$[g_{\text{Lith},l}^m, h_{\text{Lith},l}^m] = F_l'(R_1, R_2) \left\{ \begin{matrix} \xi_1 [\alpha_{l,n}^{r,m}, \beta_{l,n}^{r,m}] + \xi_2 [\alpha_{l+1,n}^{\theta,m}, \beta_{l+1,n}^{\theta,m}] \\ + \xi_3 [\alpha_{l-1,n}^{\theta,m}, \beta_{l-1,n}^{\theta,m}] + \xi_4 [-\beta_{l,n}^{\lambda,m}, \alpha_{l,n}^{\lambda,m}] \end{matrix} \right\}, \quad (10a)$$

with

$$\begin{aligned} \xi_1 &= \frac{l}{2l+1}, \quad \xi_2 = \frac{l\sqrt{(l+1)^2 - m^2}}{(2l+1)(2l+3)}, \\ \xi_3 &= -\frac{(l+1)\sqrt{l^2 - m^2}}{(2l+1)(2l-1)}, \quad \text{and } \xi_4 = \frac{m}{2l+1}, \end{aligned} \quad (10b)$$

where the coefficients $\alpha_{l,n}^{[r,\theta,\lambda],m}$ and $\beta_{l,n}^{[r,\theta,\lambda],m}$ are the SH expansion coefficients of the vector $\tilde{\psi}$ that is given in Equation (C9), and

$$F_l'(R_1, R_2) = \frac{1}{r_E^{l+2}} \int_{R_1}^{R_2} f(r') s(r') r'^{l+1} dr', \quad (10c)$$

with $f(r')$ and $s(r')$ representing the radial variation of the susceptibility and of the inducing field, respectively. Assuming now that neither the susceptibility of the magnetized lithosphere nor the inducing field vary radially, that is, $f(r') = s(r') = 1$, $F_l'(R_1, R_2)$ becomes

$$F_l'(R_1, R_2) = \left(\frac{R_2}{r_E} \right)^{l+2} \frac{1 - (R_1/R_2)^{l+2}}{l+2}. \quad (10d)$$

Alternatively, if we ignore only the radial variation of the suscepti-

bility (i.e., $f(r') = 1$) but take into account the radial variation of the induced field by setting $s(r') = (r_E/r')^{l_{\text{Core}}+2}$ for each degree l_{Core} of the core field, we obtain the Gauss coefficients of the induced lithospheric magnetic field

$$[g_{\text{Lith},l}^m, h_{\text{Lith},l}^m] = \sum_{l_{\text{Core}}=1}^{l_{\text{Core,Max}}} F_l'(R_1, R_2) \left\{ \begin{matrix} \xi_1 [\alpha_{l,n}^{r,m}, \beta_{l,n}^{r,m}] + \xi_2 [\alpha_{l+1,n}^{\theta,m}, \beta_{l+1,n}^{\theta,m}] \\ + \xi_3 [\alpha_{l-1,n}^{\theta,m}, \beta_{l-1,n}^{\theta,m}] + \xi_4 [-\beta_{l,n}^{\lambda,m}, \alpha_{l,n}^{\lambda,m}] \end{matrix} \right\}, \quad (11a)$$

where ξ_1 , ξ_2 , ξ_3 , and ξ_4 are given by Equation (10b), $\alpha_{l,n}^{[r,\theta,\lambda],m}$ and $\beta_{l,n}^{[r,\theta,\lambda],m}$ are the SH expansion coefficients of the vector $\tilde{\psi}_{l_{\text{Core}}}$ given by Equation (C9) separately for each degree l_{Core} of the core field, and

$$F_l'(R_1, R_2, l_{\text{Core}}) = \left(\frac{R_2}{r_E} \right)^{l-l_{\text{Core}}} \frac{1 - (R_1/R_2)^{l-l_{\text{Core}}}}{l-l_{\text{Core}}}. \quad (11b)$$

When $l = l_{\text{Core}}$, $F_l'(R_1, R_2, l_{\text{Core}})$ converges to $-\ln(R_1/R_2)$, which means that if a very thin spherical shell is magnetized by an inducing field of degree l_{Core} , the induced magnetic field of $l = l_{\text{Core}}$ is very weak.

2.2.2 A magnetized layer of variable thickness

Finally, we simultaneously consider the effect of an inducing field of arbitrarily high degree and that of a magnetized shell of variable thickness. As described in Section 2.2.1, the global magnetic field of the middle shell with a constant thickness can be calculated by using Equation (10) or (11). Here, we derive the necessary formulas only for the effect of the topography variations relative to the average radii, R_1 and R_2 , as these are described by $h_1(\theta', \lambda')$ and $h_2(\theta', \lambda')$, respectively.

If $f(r') = s(r') = 1$, and following a similar procedure as in Section 2.1.2, we obtain (see Supporting Text D of Supplementary Material for details)

$$[\text{[low,up]} g_{\text{Lith},l}^m, \text{[low,up]} h_{\text{Lith},l}^m] = \left(\frac{R_{[1,2]}}{r_E} \right)^{l+2} \sum_{n=1}^{l+2} \left[\frac{\prod_{j=1}^n (l+3-j)}{(l+2)n!} \right] \left\{ \begin{matrix} \xi_1 [\text{[low,up]} \alpha_{l,n}^{r,m}, \text{[low,up]} \beta_{l,n}^{r,m}] \\ + \xi_2 [\text{[low,up]} \alpha_{l+1,n}^{\theta,m}, \text{[low,up]} \beta_{l+1,n}^{\theta,m}] \\ + \xi_3 [\text{[low,up]} \alpha_{l-1,n}^{\theta,m}, \text{[low,up]} \beta_{l-1,n}^{\theta,m}] \\ + \xi_4 [-\text{[low,up]} \beta_{l,n}^{\lambda,m}, \text{[low,up]} \alpha_{l,n}^{\lambda,m}] \end{matrix} \right\}, \quad (12)$$

where ξ_1 , ξ_2 , ξ_3 , and ξ_4 are given by Equation (10b), and $\alpha_{l,n}^{[r,\theta,\lambda],m}$ and $\beta_{l,n}^{[r,\theta,\lambda],m}$ are the SH expansion coefficients of $[\text{low,up}] \tilde{\omega}_{[r,\theta,\lambda],n}(\theta', \lambda')$ given in Equations (D5) and (D6). If $f(r') = 1$ and $s(r') = (r_E/r')^{l_{\text{Core}}+2}$, we get

$$[\text{[low,up]} g_{\text{Lith},l}^m, \text{[low,up]} h_{\text{Lith},l}^m] = \left(\frac{R_{[1,2]}}{r_E} \right)^{l+2} \sum_{l_{\text{Core}}=1}^{l_{\text{Core,Max}}} \sum_{n=1}^{l+2} \left[\frac{\prod_{j=1}^n (l-l_{\text{Core}}+1-j)}{(l-l_{\text{Core}})n!} \right] \left\{ \begin{matrix} \xi_1 [\text{[low,up]} \alpha_{l,n,l_{\text{Core}}}^{r,m}, \text{[low,up]} \beta_{l,n,l_{\text{Core}}}^{r,m}] \\ + \xi_2 [\text{[low,up]} \alpha_{l+1,n,l_{\text{Core}}}^{\theta,m}, \text{[low,up]} \beta_{l+1,n,l_{\text{Core}}}^{\theta,m}] \\ + \xi_3 [\text{[low,up]} \alpha_{l-1,n,l_{\text{Core}}}^{\theta,m}, \text{[low,up]} \beta_{l-1,n,l_{\text{Core}}}^{\theta,m}] \\ + \xi_4 [-\text{[low,up]} \beta_{l,n,l_{\text{Core}}}^{\lambda,m}, \text{[low,up]} \alpha_{l,n,l_{\text{Core}}}^{\lambda,m}] \end{matrix} \right\}. \quad (13)$$

where $\alpha_{l,n,l_{\text{Core}}}^{[r,\theta,\lambda],m}$ and $\beta_{l,n,l_{\text{Core}}}^{[r,\theta,\lambda],m}$ are the SH expansion coeffi-

cients of $_{[low,up]} \omega_{[r,\theta,\lambda],n,l_{core}}(\theta', \lambda')$ for each degree l_{core} of the core field given in Equations (D5) and (D6).

3. Numerical Analysis

In this section, we utilize synthetic models to investigate the impacts of the non-axial dipolar terms, of the radial changes of the inducing field, and of the geometric shape of the magnetic lithosphere on forward calculations of the induced lithospheric magnetic field.

3.1 The Impact of the Non-axial Dipolar Terms of the Inducing Field

To investigate the extent of the impact of non-axial dipolar terms of the inducing field on forward calculations of the induced lithospheric magnetic field, we adopt a spherical shell model of constant thickness and laterally varying susceptibility. The global Vertically Integrated Susceptibility (VIS) model of Hemant and Maus (2005) is utilized as the apparent susceptibility distribution of the magnetized shell, and International Geomagnetic Reference Field 13 (IGRF13; Alken et al., 2021) at the epoch of 2005.0 is utilized as the inducing field. The thickness of the magnetized

shell is set as equal to 30 km (see Thébaud and Vervelidou, 2015, for the mean magnetic crustal thickness estimates based on a lithospheric magnetic field model). Thus, the geocentric radii of the upper and lower surfaces of the spherical shell are 6371.2 km (i.e., the radius of the geomagnetic reference sphere) and 6341.2 km, respectively. Inside the spherical shell, the average susceptibility distribution is calculated through dividing the VIS by 30 km.

By means of Equation (6), we calculate the lithospheric field when this is induced merely by the axial dipolar term of IGRF13, and by means of Equation (11), we calculate the lithospheric field when this is induced by the full expansion of IGRF13 (SH degrees from 1 to 13). The calculated fields by the full expansion of IGRF13 and the residuals between these two fields for SH degrees and orders 16–90, at 300 km altitude (left column) and at the Earth's geomagnetic reference radius (right column), are shown in Figure 2, where only the vertical component (B_z) fields are plotted.

As illustrated in Figure 2, the impact of the non-axial dipolar terms of the inducing field on the forward calculation ranges from -10.5 to $+11.7$ nT at an altitude of 300 km and from -80.1 to $+81.4$ nT on the Earth's geomagnetic reference sphere. Compared with the

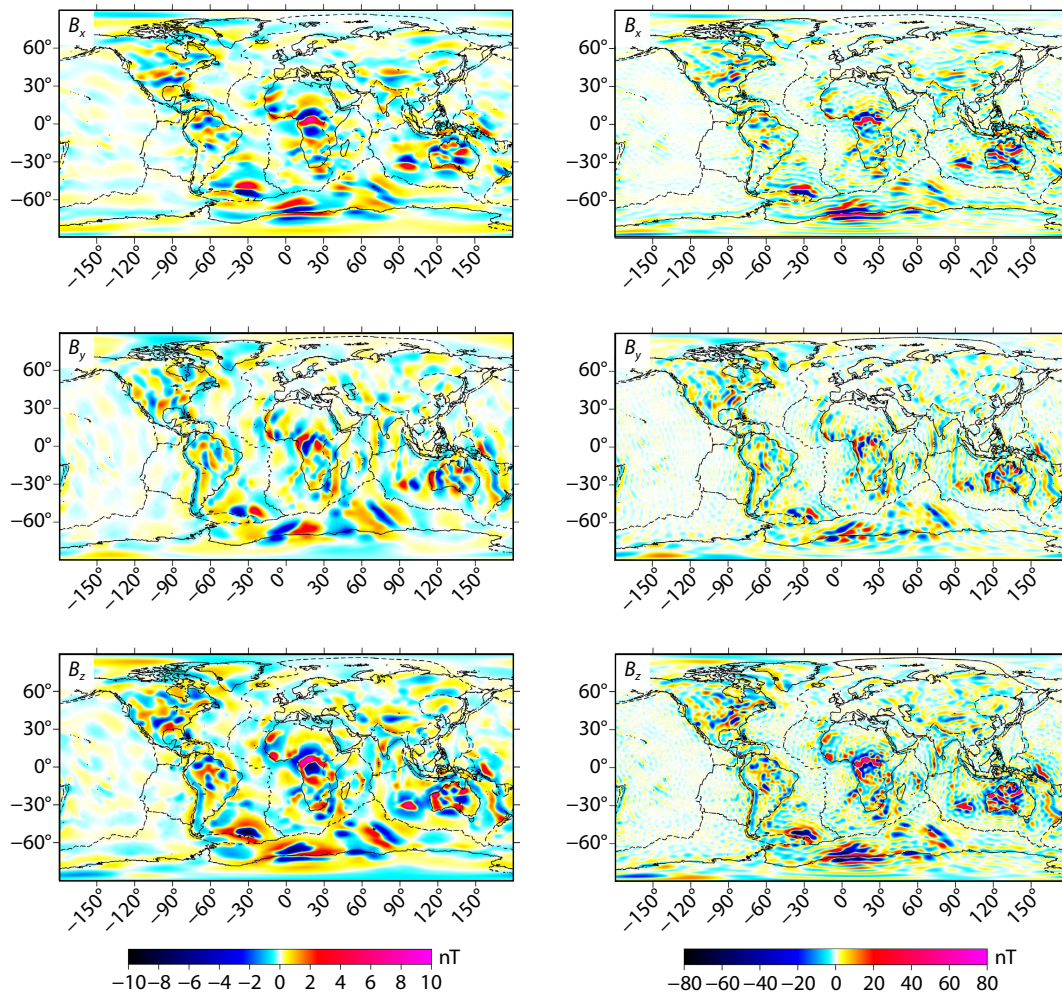


Figure 2. The impact of the non-axial dipolar terms of the inducing field on forward calculation of the global lithospheric magnetic field of SH degrees 16–90 for a 75 km thick magnetized shell, at an altitude of 300 km (left panel) and at the Earth's geomagnetic reference sphere (right panel). See text for details on the exact configuration.

total lithospheric magnetic field, which mainly varies from -15 to $+15$ nT at the altitude of 300 km and from -200 to $+200$ nT when satellite measurements are downward continued to the Earth's surface, the impact is significant. Consequently, the non-axial dipolar terms of the inducing field should not be neglected in

forward calculations of the lithospheric magnetic field. This conclusion is widely accepted, and the dipolar field approximation is not adopted any more in most studies. However, we quantify the error in this study.

The Lowes–Mauersberger power spectra of the two lithospheric

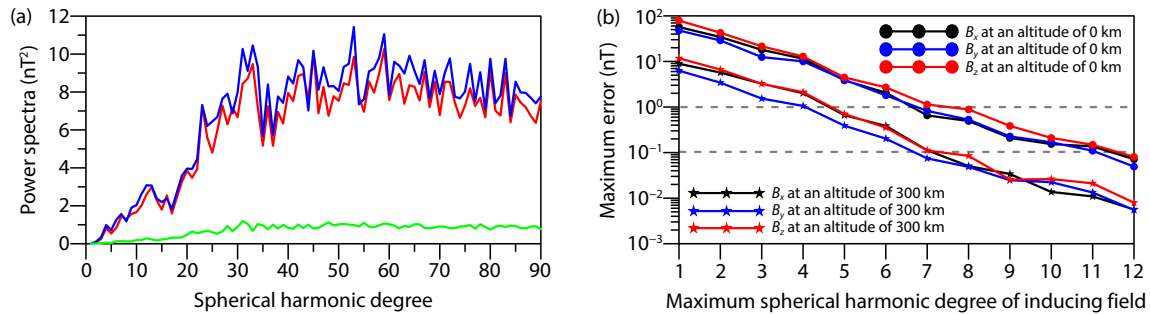


Figure 3. (a) The Lowes–Mauersberger power spectrum of the lithospheric magnetic field at the Earth's geomagnetic reference sphere, induced by an axial dipole (red line), and induced by a core field of SH degrees 1–13 (blue line). See text for details on the exact configuration. The power spectrum of their difference is also illustrated (green line). (b) The maximum error, with respect to considering an inducing field of SH degrees 1–13, as a function of the maximum SH degree of the inducing field, at an altitude of 300 km and at the Earth's geomagnetic reference sphere. The results are shown separately for each magnetic field component.

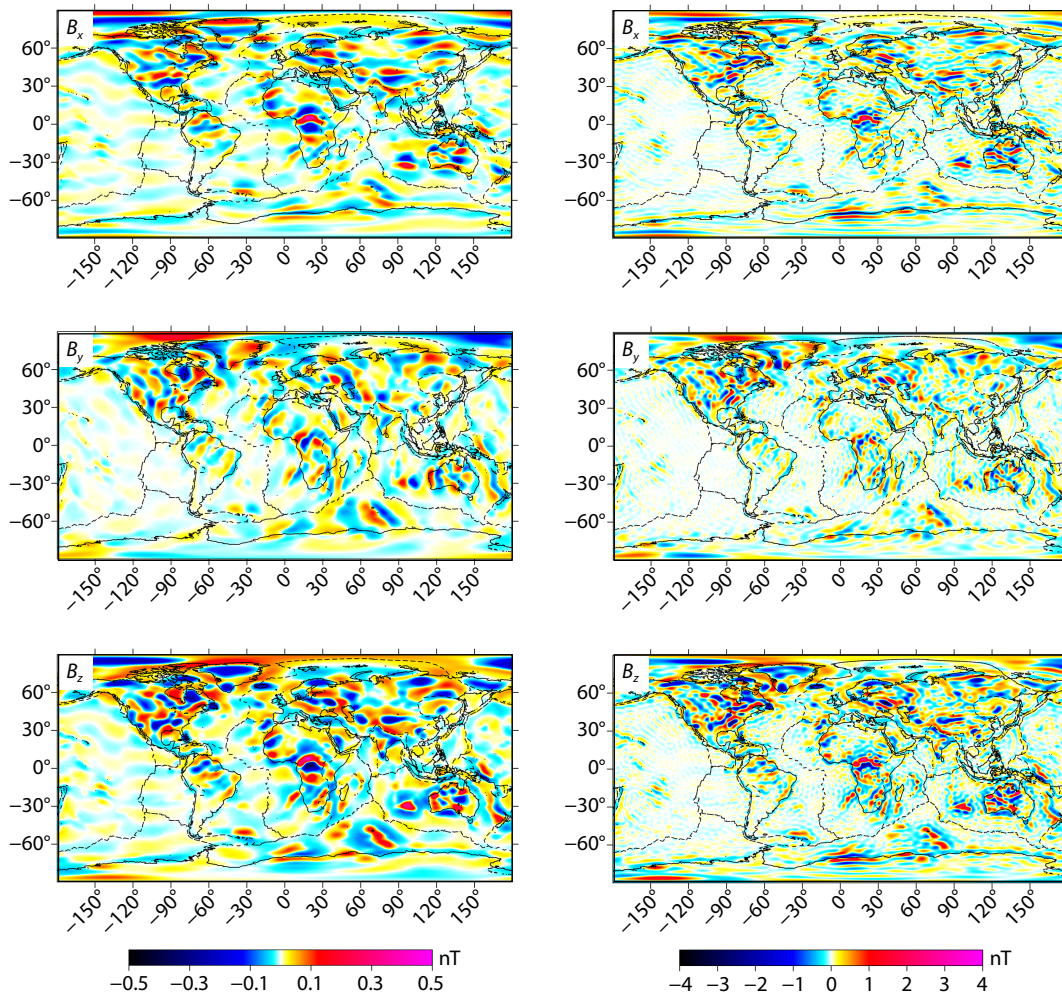


Figure 4. The impact of the radial variation of the inducing field on forward calculation of the global lithospheric magnetic field of SH degrees 16–90, at an altitude of 300 km (left panel) and at the Earth's geomagnetic reference sphere (right panel). See text for details on the exact configuration.

fields (Lowes, 1974) are shown in Figure 3a. The difference of the two spectra, in both energy level and shape, becomes clear after approximately SH degree 20. The small wavelengths of the lithospheric field are affected more than its large wavelengths. This effect was also suggested by Arkani-Hamed and Dymant (1996) (see their Figure 4).

To quantitatively evaluate this effect for different maximum SH degrees of the inducing field, we plot in Figure 3b the maximum error (i.e., the maximum value) and the standard deviation of the residuals between the lithospheric field induced by the full expansion of the IGRF13 (maximum SH degree 13) and the lithospheric field induced by subsets of the IGRF13. As shown in this figure, for a maximum error lower than 0.1 nT (1 nT) at 300 km altitude (at the Earth's surface), the inducing field should be considered up to at least SH degree 7.

3.2 The Impact of the Radial Changes of the Inducing Field

To evaluate the effect of neglecting its radial variation when forward modelling the lithospheric field, we once more utilize the global magnetization model described in Section 3.1. From the Earth's surface down to a depth of 75 km, the intensity and the inclination of the inducing field change by less than 6% and $\pm 0.8^\circ$, respectively, and for the declination, the changes are smaller than $\pm 1^\circ$, except for the areas around the geomagnetic poles (see Figure S1). Utilizing Equations (10) and (11), we calculate the lithospheric field induced by the full expansion of IGRF13 when the radial variation of the inducing field is neglected and when it is taken into account, respectively. In the first case, the inducing field is calculated at the top of the magnetic layer. The differences for SH degrees 16–90 between the two results are calculated and plotted in Figure 4 at the altitude of 300 km (left column) and on the Earth's geomagnetic reference sphere (right column), respectively. According to this figure, the radial variation of the inducing field produces a maximum impact of ± 0.5 nT at 300 km altitude and of ± 4 nT at the Earth's geomagnetic reference radius, respectively. This result suggests that, for most applications, the radial variation of the inducing field on forward calculations of the large-scale lithospheric magnetic field can be neglected.

The power spectra of the two lithospheric fields and of their resid-

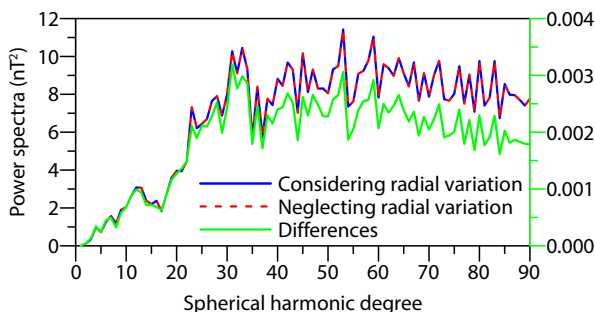


Figure 5. The Lowes–Mauersberger power spectrum of the induced lithospheric magnetic field at the Earth's geomagnetic reference sphere, when the radial variation of the inducing field is considered (blue line), or is neglected (red line). The power spectrum of their difference is scaled by the right axis (green line). See text for details on the exact configuration.

uals are shown in Figure 5. According to this figure, despite raising with increasing SH degree, this effect is very weak. Therefore, in Section 3.3, this effect is neglected. However, to mitigate its impact, the inducing field is calculated at the middle of the spherical shell rather than at its top, as described in this section. The impact of this modification in the spatial and spectral domains is shown in Figures S2 and S3, respectively. In this case, the radial variation of the inducing field produces a maximum impact of ± 0.03 nT at 300 km altitude and of ± 0.5 nT at the Earth's geomagnetic reference sphere, respectively. These results suggest that the effect of neglecting the radial variation of the inducing field on forward calculations is significantly suppressed merely by considering the middle of the magnetic layer as the reference surface of the inducing field.

3.3 The Impact of the Geometric Shape of the Magnetic Lithosphere

In this section, we want to assess the extent to which the lithospheric magnetic fields of two spherical shell models with different geometric shapes but with identical vertically integrated susceptibility distributions differ from each other.

Since Wasilewski and Mayhew (1992) first suggested that the Moho discontinuity is also a magnetic boundary, some studies have shown that the uppermost mantle can be locally magnetic (e.g., Blakely et al., 2005; Ferré et al., 2014, 2021). However, Vervelidou and Thébaud (2015) showed that on a global scale, the average of the Moho depth and of the magnetic thickness coincide (see their Figure 12; note that the unit of the y axis is actually square kilometers). Furthermore, on the basis of these considerations, we make use of the CRUST1.0 model (Laske et al., 2013) and adopt the Moho interface and the bottom of the sediment layer as the lower and upper surface of the magnetic lithosphere, respectively. The average geocentric radii of these two interfaces are $R_1 = 6349.68$ km and $R_2 = 6367.67$ km, which gives an average thickness of $d = 17.99$ km. The topography distributions relative to these two spheres (i.e., h_1 and h_2), and the thickness variation (i.e., $r_2 - r_1$) are shown in Figure S4. It is worth noting, that the topography in Figure S4 is dominated by the Earth's flattening. This is due to the conversion of CRUST1.0 from geodetic to geocentric coordinates. The magnetic effect of the Earth's ellipticity has been studied by Jackson et al. (1999) as well as Lesur and Jackson (2000) and has been found to be very weak. By dividing the VIS (Figure S5a) by the variable thickness of this layer, the susceptibility distribution is obtained (Figure S5b). As in previous studies, the inducing field is taken to be IGRF13 (Alken et al., 2021) at the epoch of 2005.0.

By means of Equation (12), we calculate the lithospheric field generated by this magnetized layer of variable thickness and compare it with the lithospheric field generated by two magnetized layers, one with $d = 17.99$ km ($R_1 = 6353.21$ km and $R_2 = 6371.2$ km) and one with $d = 75$ km ($R_1 = 6296.2$ km and $R_2 = 6371.2$ km).

Concerning the power series expansion of the relative topography, that is, of the term $\frac{(1 + h_{[1,2]}/R_{[1,2]})^{l+2} - 1}{l+2}$ (see Equation (D3)), here we take into account the terms up to only the 5th power. Figure S6 shows the contribution to the lithospheric magnetic

field power spectrum of the expansion terms with $n = 1$ up to $n = 5$. According to this figure, the contribution of the term with $n = 5$ is already negligible. In the spatial domain, the maximum absolute amplitude of this term is 0.01 nT at 300 km altitude and 0.3 nT at the Earth's geomagnetic reference sphere.

The residuals are plotted in Figures 6 and 7, respectively. The residuals range from nearly -3.9 to $+5.8$ nT (left panel in Figure 6) and -3.2 to $+5.6$ nT (left panel in Figure 7) at 300 km altitude and from -41.4 to $+59.3$ nT (right panel in Figure 6) and -34.5 to $+48.3$ nT (right panel in Figure 7) at the Earth's geomagnetic reference radius. The Lowes–Mauersberger power spectra of the three models considered here and of their residuals are illustrated in Figure 8a. It should be noted that these three models have the same vertically integrated susceptibility distribution, which justifies the observation that the thinner the magnetic layer is, the more energetic its power spectrum. Figures 6, 7, and 8a suggest that the effect of the geometric shape of the magnetized shell is significant even at the satellite altitude.

4. Discussion

The effect of the magnetic thickness for a given vertically inte-

grated susceptibility distribution becomes clearer by comparing the spectra of these three models with the spectrum of the global lithospheric magnetic field model MF7 (updated version of the MF6 model; Maus et al., 2008) (see Figure 8, black line).

As illustrated in Figure 8a, we observe that the synthetic models are less energetic than MF7. In this respect, we note that Hemant and Maus (2005) reported that to minimize the root mean square difference between the Gauss coefficients of the MF3 model (Maus et al., 2006) and the magnetic field predicted by their VIS model, they had to scale the average maximum susceptibility values of all rock types by a factor of 0.55. We think that the need for this scaling is caused by the impact of the geometric shape of the magnetic lithosphere because they considered an infinitesimally thin layer on the Earth's surface. We suggest that the finite thickness model of the magnetic lithosphere employed in our study renders this scaling factor redundant. To test this suggestion, we multiply the VIS model (Hemant and Maus, 2005) by $1.0/0.55$ and then re-calculate the magnetic fields of the three synthetic models described above. Now, as shown in Figure 8b, the Lowes–Mauersberger power spectra of the three synthetic models are very close to that of MF7, which implies that our

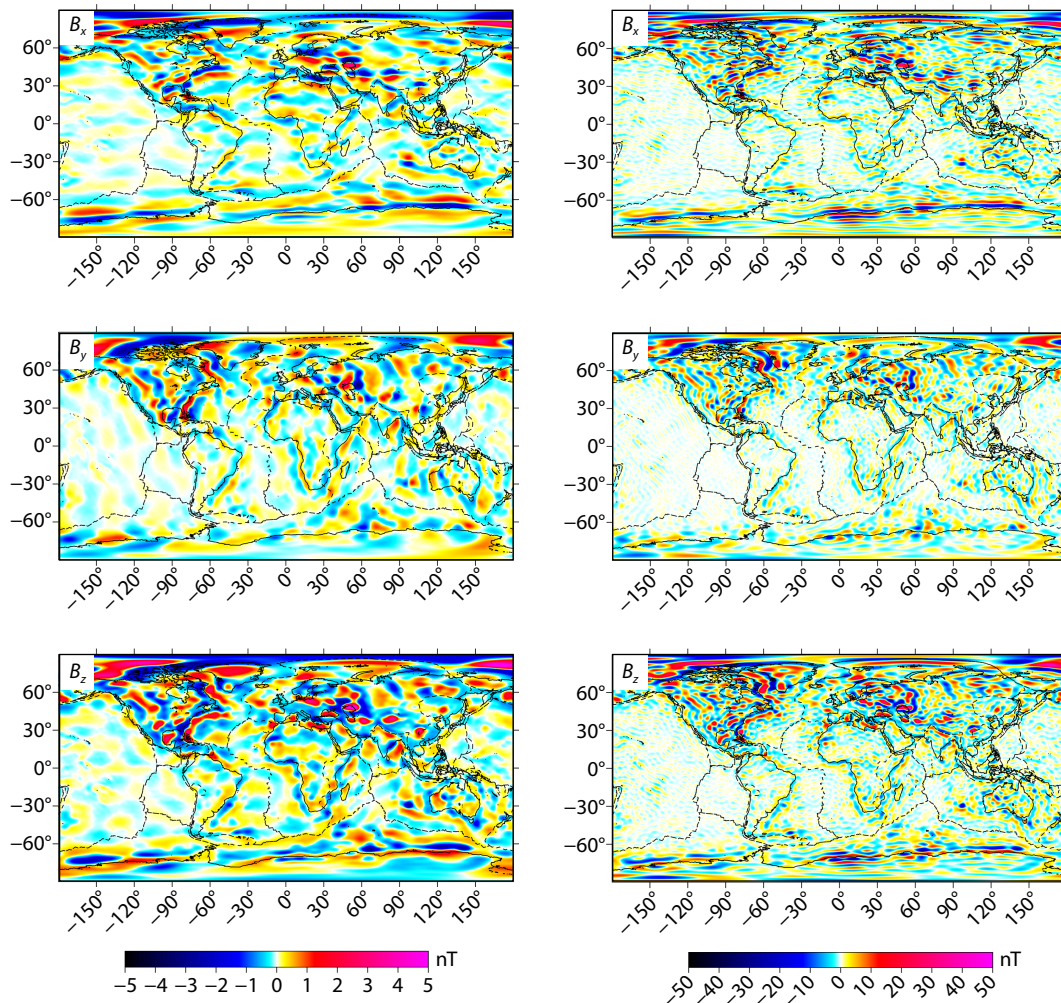


Figure 6. The impact of the magnetic shell's geometric shape on forward calculation of the global lithospheric magnetic field of SH degrees 16–90, at an altitude of 300 km (left panel) and at the Earth's geomagnetic reference sphere (right panel), when $d = 17.99$ km. See text for details on the exact configuration.

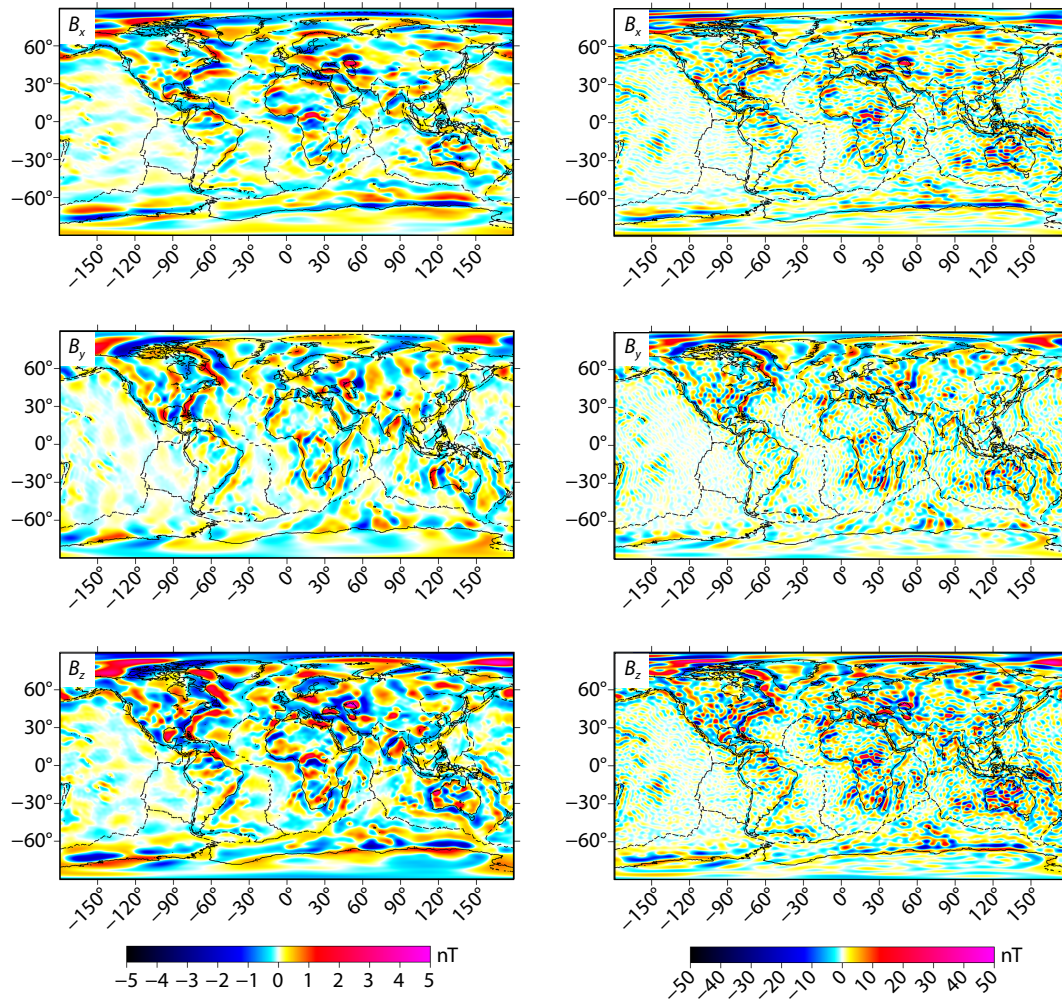


Figure 7. The same as Figure 6 but for $d = R_2 - R_1 = 75$ km. See text for details on the exact configuration.

suggestion is correct. A more detailed comparison between the three synthetic models and the MF7 model is beyond the scope of this study.

5. Conclusions

In this study, we present analytical formulae for the SH coefficients of the induced lithospheric magnetic field for an arbitrarily high maximum expansion degree of the inducing field as well as for a magnetic shell of variable thickness. On the basis of the derived formulae, we perform several synthetic tests, from which the following three main conclusions are drawn:

(1) Ignoring the non-axial dipolar terms of the inducing field produces an error in the forward calculation of the global lithospheric magnetic field of SH degrees 16–90, which is approximately ± 10 nT at the typical low earth orbit satellite altitude of 300 km and approximately ± 80 nT at the Earth's geomagnetic reference sphere, for a 75 km thick magnetic shell. We conclude that neglecting the non-axial dipolar terms of the inducing field significantly reduces the accuracy of forward calculation of the lithospheric magnetic field.

(2) The impact of the radial variation of the inducing field on forward calculation of the global lithospheric magnetic field of SH degrees 16–90, is approximately ± 0.5 nT at the typical low earth

orbit satellite altitude of 300 km and ± 4 nT at the Earth's geomagnetic reference radius, for a 75 km thick magnetic shell. We conclude that neglecting this effect does not introduce a significant error in forward calculation of the large scales of the lithospheric magnetic field. However, the effect is further suppressed when the reference surface of the inducing field is taken to be the middle of the magnetized layer.

(3) The geometric shape of the Earth's magnetic lithosphere has a strong impact on the induced lithospheric magnetic field. The lithospheric fields generated by two spherical shell models with different thicknesses but with identical vertically integrated susceptibility distributions are different even at satellite altitudes. In particular, underestimating the thickness of the magnetized layer overestimates the induced lithospheric magnetic field. The extreme case of considering the magnetic lithosphere to be an infinitesimally thin layer either yields unrealistically large magnetic field values or leads to an underestimation of the susceptibility values. In this respect, our numerical analysis indicates that the underlying scaling factor of 0.55 in the VIS model of Hemant and Maus (2005) is due to the impact of the geometric shape and thickness of the magnetic lithosphere and in particular to their modelling of the magnetic lithosphere as an infinitesimally thin layer at the Earth's surface. Modelling the magnetic litho-

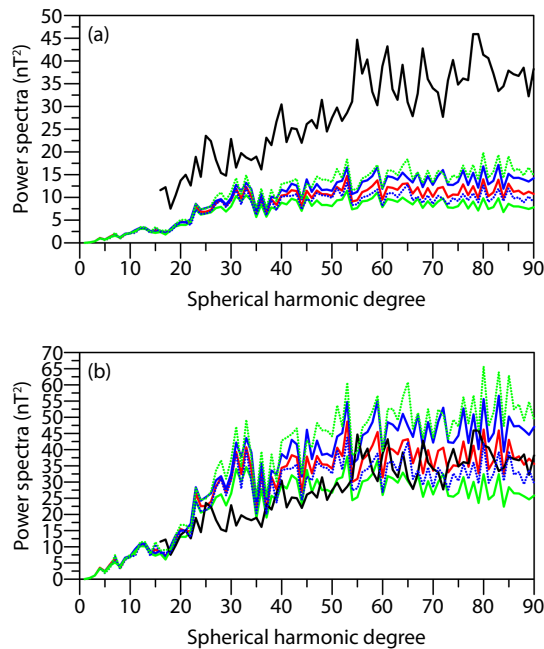


Figure 8. The Lowes–Mauersberger power spectrum of the induced lithospheric magnetic field at the Earth’s geomagnetic reference radius, for a 17.99 km thick magnetized shell (solid blue line), for a 75 km thick magnetized shell (solid green line), and for a magnetized layer with the thickness distribution shown by Figure S4c (solid red line). The power spectra of the differences between the two shells of constant thickness and the shell of variable thickness are also shown (dashed blue and green lines for shells of constant thickness with $d = 17.99$ km and $d = 75$ km, respectively). The power spectrum of the MF7 is also shown (solid black line). In (a), the global VIS model (Hemant and Maus, 2005) is used, but in (b), the global VIS distribution is multiplied by a constant factor of 1.0/0.55. See text for details on the exact configuration.

sphere as a layer of finite thickness removes the need for this scaling factor. This result indicates that the susceptibility values measured in laboratories are in good agreement with the susceptibility values required to explain the observed magnetic field.

Our study, thanks to the insights it provides concerning the sensitivity of lithospheric magnetic fields at satellite altitudes and the formulae of Section 2.2.2, opens the possibility for more accurate estimations of the apparent susceptibility distribution (e.g., Vervelidou and Lesur, 2018; Lesur and Vervelidou, 2020) and the depth to the bottom variations of the magnetic lithosphere, in combination with the available global lithospheric magnetic field models (e.g., Olsen et al., 2017; Sabaka et al., 2020; Thébaud et al., 2021; Jiang Y et al., 2024) and the special models derived from the CSES-1 (e.g., Shen XH et al., 2018), MSS-1 (e.g., Zhang K, 2023) and the upcoming MSS-2 (e.g., Jiang Y et al., 2023; Olsen et al., 2023; Livermore et al., 2024) satellite magnetic measurements. Moreover, by means of the formulae presented in this article, the “hidden” long-wavelength components of the global lithospheric magnetic field can be inferred (see Vervelidou and Thébaud, 2015, concerning the hidden long wavelengths of the magnetic thickness).

Supplementary Material

Additional Supplementary Material can be found in the online version of this article.

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