

An alternative regularization approach for modeling the regional lithospheric magnetic field from satellite magnetic measurements by the spherical equivalent source method and its application in Africa

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Key Points:

- Both the minimum and flattest models were introduced to regularization in the spherical equivalent source method, effectively suppressing the Backus effect and mitigating inversion instability in the low-latitude regions.
- An equivalent source model of the large-scale lithospheric magnetic field over Africa was built from Macau Science Satellite-1 (MSS-1) and China Seismo-Electromagnetic Satellite-1 (CSES-1) magnetic measurements.
- The derived susceptibility contrast distribution correlates well with the regional tectonic framework in Africa and surroundings.

Citation: Gui, J. S., Du, J. S., Zhang, P., Yuan, C. Q., Li, Y. K., Zhao, K. A., Yin, L., and Li, H. P. (2025). An alternative regularization approach for modeling the regional lithospheric magnetic field from satellite magnetic measurements by the spherical equivalent source method and its application in Africa. *Earth Planet. Phys.*, 9(3), 697–710. <http://doi.org/10.26464/epp2025066>

Abstract: The equivalent source (ES) method in the spherical coordinate system has been widely applied to processing, reduction, field modeling, and geophysical and geological interpretation of satellite magnetic anomaly data. However, the inversion for the ES model suffers from nonuniqueness and instability, which remain unresolved. To mitigate these issues, we introduce both the minimum and flattest models into the model objective function as an alternative regularization approach in the spherical ES method. We first present the methods, then analyze the accuracy of forward calculation and test the proposed ES method in this study by using synthetic data. The experimental results from simulation data indicate that our proposed regularization effectively suppresses the Backus effect and mitigates inversion instability in the low-latitude region. Finally, we apply the proposed method to magnetic anomaly data from China Seismo-Electromagnetic Satellite-1 (CSES-1) and Macau Science Satellite-1 (MSS-1) magnetic measurements over Africa by constructing an ES model of the large-scale lithospheric magnetic field. Compared with existing global lithospheric magnetic field models, our ES model demonstrates good consistency at high altitudes and predicts more stable fields at low altitudes. Furthermore, we derive the reduction to the pole (RTP) magnetic anomaly fields and the apparent susceptibility contrast distribution based on the ES model. The latter correlates well with the regional tectonic framework in Africa and surroundings.

Keywords: regional lithospheric magnetic field model; spherical equivalent source method; satellite magnetism; Africa; MSS-1; CSES-1

1. Introduction

The lithospheric magnetic field is a key component of the geomagnetic field. It reflects the spatial distribution of magnetization in the lithosphere, arising from both induced and remanent

magnetization in the crust and uppermost mantle (Ferré et al., 2021). The lithospheric magnetization variations can provide valuable insights into geological structure and geothermal architecture (Hemant and Maus, 2005; Purucker and Clark, 2011; Purucker and Whaler, 2015; Liu PF et al., 2023). Satellite magnetic survey data have wide global coverage, making the data suitable for large-scale lithospheric magnetic field modeling. Compared with ground and airborne surveys, satellite observations are less influenced by local topography variations, human activities, and shallow magnetized sources with small scales, making them ideal for

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Received 17 DEC 2024; Accepted 24 MAR 2025.
First Published online 21 APR 2025.
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studying regional tectonics, as well as lithospheric thermal and rheological frameworks (Wang J et al., 2024a, b).

Lithospheric magnetic field models derived from satellite data commonly utilize harmonic analysis methods or equivalent source (ES) methods. The former methods include SHA (spherical harmonic analysis), RHA (rectangular harmonic analysis), SCHA (spherical cap harmonic analysis), and R-SCHA (revised spherical cap harmonic analysis), all of which generally achieve high accuracy. Rectangular harmonic analysis is the most effective for small areas, whereas SCHA and R-SCHA are effective for regional models, and SHA is effective for global models (Xu WY et al., 2011). Alternatively, in the spatial domain, the ES method proposed by Dampney (1969) utilizes fictitious sources to fit observational data and requires substantial computer resources to achieve high accuracy and flexibility for diverse data types and data processing objectives. Notably, the ES method can generate a magnetization model that contributes to geophysical and geological interpretation; it can also perform the reduction to the pole (RTP; von Frese et al., 1986). Therefore, the ES method in a spherical coordinate system has previously been applied to process the satellite magnetic anomaly data (e.g., Mayhew, 1979, 1982; Mayhew et al., 1980; von Frese et al., 1981).

However, Mayhew et al. (1985) noted that the ES method exhibits instability in mathematical inversion, particularly near the magnetic equator. The underlying causes remain unclear. The most probable explanation is the Backus effect (Backus, 1970), which was recently highlighted by Luo Y et al. (2024). Backus (1970) emphasized that the scalar magnetic data alone cannot determine the vector or three components of the magnetic field. In the forward and inverse calculations of the scalar magnetic anomaly data (i.e., the total magnetic intensity [TMI] anomaly, or the so-called ΔT magnetic anomaly), the equation

$$\text{TMI} = B_x \cos I \cos D + B_y \cos I \sin D + B_z \sin I$$

is typically adopted, where B_x , B_y , and B_z are the three components along the north, east, and downward directions respectively, and I and D are the inclination and declination of the main magnetic field, respectively. This approach presents two main issues. First, discrepancies exist between the projection of the three components onto the main magnetic field and the scalar data (Yuan XY et al., 2015), leading to poor fitting of the scalar data in certain regions. Second, although the projection of the three components onto the main magnetic field can be accurately derived from the TMI data (e.g., Sun SD et al., 2020; Hu ZW et al., 2021), reconstructing the three components solely from TMI data remains challenging. Particularly in low magnetic latitudes, the geomagnetic inclination will be close to zero, and the B_z component will not participate in the synthesis, resulting in a very weak constraint on the B_z component in the inversion. As a result, inversion instability arises in the ES method, producing oscillations, particularly in low magnetic latitude regions near the magnetic equator. Mayhew (1979) and Mayhew et al. (1980) initially mitigated this instability by using larger dipole spacing at low latitudes and smaller spacing at high latitudes to achieve stable solutions. von Frese et al. (1988) applied the ridge regression to suppress the instability. Purucker (1990) proposed utilizing a technique such as singular value

decomposition and incorporating both scalar and appropriately weighted vector data to stabilize the ES solution. Purucker et al. (1998) integrated extensive seismic, geological, and rock magnetic data into the initial model during the inversion process, and Whaler (1994), Whaler and Langel (1996), and Whaler and Purucker (2005) constrained model complexity (i.e., the regularization term) by using a minimum model in the L_2 norm. The minimum model aims to minimize the total energy of the inverted magnetic susceptibility, thereby reducing nonuniqueness. However, exclusively applying a globally uniform minimum model has resulted in drastic variations in the amplitude and direction of the inverted magnetization, leading to significant deviations from actual conditions (Whaler, 2007).

In this study, we propose an ES method in the spherical coordinate system in which we integrate a regularization technique to balance the minimum model, flattest model, and data fitting errors. This approach effectively mitigates the Backus effect, reducing inversion instability in the magnetic equatorial region, as demonstrated with a case study of Africa. Section 2 introduces the modeling methods, detailing the setup of the spherical ES model, as well as the forward and inversion techniques utilized. In Section 3, we verify the reliability of the proposed method by using simulated lithospheric magnetic field data, ensuring its applicability to practical data. Section 4 outlines the practical dataset used to establish the lithospheric magnetic field model, which primarily consists of geomagnetic measurements from the Macau Science Satellite-1 (MSS-1) and China Seismo-Electromagnetic Satellite-1 (CSES-1). Section 5 presents the results, with emphasis on the reliability evaluation and downward continuation outcomes from the established model. Section 6 examines the impact of different length scales and compares the new lithospheric field model with existing models. Finally, Section 7 concludes the study and offers some insights into further improvements and potential applications of the study.

2. Method

2.1 Spherical ES Model Setting

The setup of the spherical ES primarily considers three aspects: the depth, the element type, and the arrangement of the spherical ES. First, regarding depth, the average global lithospheric magnetic thickness is 36 km (Thébault et al., 2021). If tesserooids (spherical prisms) are used, their thickness should be set to 36 km. However, because magnetic dipoles are being used instead, they are placed at the geometric center of the tesseroid, meaning the global magnetic dipoles are positioned at a depth of 18 km below the CGCS2000 (China Geodetic Coordinate System 2000) ellipsoid surface, as shown in Figure S1(a). Second, regarding the sparsity and arrangement of the spherical ES, a test is performed using a single magnetic dipole, which is set at (0° , 0°) at a depth of 18 km below the CGCS2000 ellipsoid surface. The parameters of the dipole are set as follows: inclination and declination of 0° , susceptibility of 0.05 SI, and induced field strength of 50,000 nT.

The horizontal attenuation characteristics at altitudes of 500 and 0 km along the 0° meridian are calculated, as shown in Figure S2. It is evident that the horizontal attenuation occurs more rapidly if

the observation altitude is lower. In this study, we consider upward continuation to an altitude of 500 km; therefore, expanding the modeling region outward by 35° is deemed reasonable. For the area extended by 35° beyond the study region [55°W–90°E, 70°S–75°N], a grid of 0.5° × 0.5° is used, with 0.5° × 0.5° × 36 km tesserooids placed and replaced by magnetic dipoles at their geometric centers. Likewise, considering the influence of the global lithosphere, the remaining area (excluding [55°W–90°E, 70°S–75°N]) is covered with a grid of 1° × 1° × 36 km tesserooids, which are also replaced by magnetic dipoles at their geometric centers. The global ES layer is shown in Figure S1(b).

The reason for replacing the 0.5° × 0.5° × 36 km tesserooids with magnetic dipoles in the study area is that, at the satellite altitudes of MSS-1 (average altitude of 450 km) and CSES-1 (average altitude of 500 km), the relative error introduced by using magnetic dipoles as replacements is less than 1% in the equatorial and midlatitude regions, and it is less than 2% in the polar regions (no setup is applied for polar regions in this study). The results are shown in Figure S3. Additionally, the tesserooids can be approximated by the magnetic dipoles to improve calculation efficiency at high altitudes. Furthermore, during downward continuation calculations, the tesserooids can be used directly for high-accuracy predictions via the Gauss–Legendre integration.

2.2 Forward Calculation Method

This study utilizes magnetic dipoles for field modeling, so the forward calculation is based on the equation for magnetic dipoles in the spherical coordinate system (Dyment and Arkani-Hamed, 1998). However, during downward continuation, the use of magnetic dipoles will lead to an unreal increase in forward values and significant errors. Therefore, forward calculations at lower altitudes must be performed using an adaptive Gauss–Legendre integration technique (Du JS et al., 2015; Yuan CQ et al., 2025) to obtain a reliable solution with higher accuracy, especially the example in Section 3.3 at an altitude of 20 km.

2.3 Inverse Calculation Method

This study utilizes an inversion method based on the L_2 norm for magnetic dipoles in the spherical coordinate system. The inversion objective function $\phi(\mathbf{m})$ consists of two parts: the data misfit function ϕ_d and the model objective function ϕ_m . Its expression is as follows:

$$\phi(\mathbf{m}) = \phi_d + \beta\phi_m, \quad (1)$$

where β is the regularization parameter, used to balance the data misfit function ϕ_d and the model objective function ϕ_m , that is, to balance the data fitting and the complexity of the ES model.

The expression for the data misfit function ϕ_d is as follows:

$$\phi_d = \sum_{l=1}^N \left(\frac{d_l^{\text{pre}} - d_l^{\text{obs}}}{\varepsilon_l} \right)^2. \quad (2)$$

Rewritten in matrix form, it is

$$\phi_d = \left\| \mathbf{W}_d(\mathbf{d}^{\text{pred}} - \mathbf{d}^{\text{obs}}) \right\|_2^2, \quad (3)$$

$$\mathbf{W}_d = \text{diag}(1/\varepsilon_l), \quad (4)$$

where $d_l^{\text{pre}}, d_l^{\text{obs}}, \varepsilon_l$ represent the model prediction value, observation value, and standard deviation of the observation errors, respectively, for the l th observation point.

For the model objective function ϕ_m , its expression is as follows:

$$\phi_m = \alpha_s \sum_{i,j} m_{ij}^2 + \alpha_\varphi \sum_{i,j} (\Delta_\varphi m_{ij})^2 + \alpha_\lambda \sum_{i,j} (\Delta_\lambda m_{ij})^2, \quad (5)$$

where α_s is the global minimizing factor (minimum model coefficient), and $\alpha_\varphi, \alpha_\lambda$ are the smoothing factors in the latitude and longitude directions (flattest model coefficients), respectively. In addition, m_{ij} is the magnetization or magnetic intensity amplitude of each cell to be inverted, and $\Delta_\varphi, \Delta_\lambda$ are the differential operators in the latitude and longitude directions, respectively, and are defined as follows:

$$\begin{cases} \Delta_\varphi f(i, j) = \frac{f(i+1, j) - f(i, j)}{\text{arc length}_{i+1, i}}, \\ \Delta_\lambda f(i, j) = \frac{f(i, j+1) - f(i, j)}{\text{arc length}_{j+1, j}}, \end{cases} \quad (6)$$

where $\text{arc length}_{i+1, i}, \text{arc length}_{j+1, j}$ represent the spherical arc distances between the previous and next cells in the latitude and longitude directions, respectively.

Equation (5) can be discretized and expressed in matrix form as

$$\phi_m = \left\| \mathbf{W}_s \mathbf{m} \right\|_2^2 + \left\| \mathbf{W}_\varphi \mathbf{m} \right\|_2^2 + \left\| \mathbf{W}_\lambda \mathbf{m} \right\|_2^2 = \left\| \mathbf{W}_m \mathbf{m} \right\|_2^2, \quad (7)$$

$$\mathbf{W}_m^T \mathbf{W}_m = \mathbf{W}_s^T \mathbf{W}_s + \mathbf{W}_\varphi^T \mathbf{W}_\varphi + \mathbf{W}_\lambda^T \mathbf{W}_\lambda. \quad (8)$$

Equations (1), (3), and (7) can then be combined to obtain the inversion objective function $\phi(\mathbf{m})$. The $\mathbf{d}^{\text{pred}}$ can be replaced with Equation (9) as

$$\mathbf{d}^{\text{pred}} = \mathbf{G} \mathbf{m}, \quad (9)$$

$$\phi(\mathbf{m}) = \left\| \mathbf{W}_d(\mathbf{G} \mathbf{m} - \mathbf{d}^{\text{obs}}) \right\|_2^2 + \beta \left\| \mathbf{W}_m \mathbf{m} \right\|_2^2, \quad (10)$$

where $\mathbf{G}$ is a kernel matrix in the forward computation.

The partial derivative of $\mathbf{m}$ can be taken and set to 0 to solve for the minimum value of $\phi(\mathbf{m})$. The result is the following system of linear equations:

$$\mathbf{A} \mathbf{m} = \mathbf{B}, \quad (11a)$$

$$\begin{cases} \mathbf{A} = \mathbf{G}^T \mathbf{W}_d^T \mathbf{W}_d \mathbf{G} + \beta \mathbf{W}_m^T \mathbf{W}_m, \\ \mathbf{B} = \mathbf{G}^T \mathbf{W}_d^T \mathbf{W}_d \mathbf{d}. \end{cases} \quad (11b)$$

Finally, we use the conjugate gradient method to solve the system of linear equations in Equation (11), where we adjust the regularization parameter β through iterative tests, using the L-curve method to achieve an optimal balance between model flatness and data misfit (Hansen, 1998).

In the model objective function, the relative weight between the minimum model objective function and the flattest model objective function is carried out by the value of the coefficient α , where α_s corresponds to the minimum model objective function, and α_φ and α_λ relate to the flattest model objective functions in the latitude and longitude directions, respectively. If the ratio (i.e., α_φ/α_s or α_λ/α_s) is too low, the resulting model will lose flatness in the reconstruction. Therefore, it is important to determine a reasonable

value for the weight coefficients α . Here, the length scales defined by Oldenburg et al. (2005) are introduced as

$$L_\varphi = \sqrt{\frac{\alpha_\varphi}{\alpha_s}}, \quad (12a)$$

$$L_\lambda = \sqrt{\frac{\alpha_\lambda}{\alpha_s}}. \quad (12b)$$

Without any prior information, Oldenburg et al. (2005) suggested that the size of the length scale should be determined based on the grid size used to discretize the subsurface space. As a result, different ES sizes and layout methods have different length scales, making the selection of the length scale a topic of discussion.

3. Synthetic Data Test

3.1 Simulation of the Magnetic Lithosphere

To test the validity of the method proposed in this study, it is essential to simulate a realistic magnetic lithosphere as accurately as possible. A global lithospheric magnetization model with a resolution of $0.25^\circ \times 0.25^\circ$ was constructed by combining the oceanic remanent magnetization model of Masterton et al. (2013) and the induced magnetization model of Hemant and Maus (2005). The thickness of the magnetic lithosphere was determined by using the Earth crustal model 1 (ECM1; Mooney et al., 2023) to calculate the depths of the Moho and the base of the sedimentary layer. The depth of the Moho was taken as the lower boundary of the magnetic lithosphere, whereas the depth of the base of the sedimentary layer was treated as the upper boundary. Because of the very low magnetic susceptibility of the regional sedimentary layer and seawater, their magnetism could be ignored (e.g., Clark, 1997; Hinze et al., 2013). Thus, as a “hard” constraint, the base of the sedimentary layer was enforced as the upper boundary of the inversion area. Although increasing evidence suggests that magnetism may also exist at the base of the uppermost mantle (e.g., Blakely et al., 2005; Ferré et al., 2021), some scholars argue that the magnetic boundary does not exceed beyond the Moho (e.g., Vervelidou and Thébault, 2015). Here, the Moho was chosen as the lower boundary, which is more representative of the actual situation. Ultimately, this allowed us to derive a magnetic structural model of the lithosphere that closely approximates the actual situation, which can be used for subsequent experiments. The related datasets adopted for simulating the magnetic lithosphere model in Africa and its surroundings are shown in Figure S4.

3.2 Simulating Magnetic Data

For the magnetic lithosphere model established in Section 3.1, the forward modeling of satellite magnetic anomaly data was performed using the forward modeling method to obtain greater accuracy.

Because the directly forward modeled magnetic field signals include spherical harmonic (SH) components of degree 15 and below, whereas the processed satellite magnetic lithosphere signals contain components of only degree 16 and above (i.e., magnetic anomaly signals), it is necessary to perform SH expansion on the forward modeling data to remove the background components of degree 15 and below, thereby closely simulating the actual processed data. The method involves forward modeling

the B_z component at an altitude of 500 km and then performing SH expansion (Rexer and Hirt, 2015) to extend it to the same degrees as the LCS-1 model (maximum degree 185) (Olsen et al., 2017), hereafter referred to as SH185.

To simulate practical magnetic anomaly data, noise must be added to the forward modeling data because the measured data inherently contain noise. Here, the random Gaussian noise with a mean of 0 nT was generated by using the fitting function of the noise standard deviation reported in Section 4. The noise was then added to the forward modeling data to obtain the simulated satellite magnetic anomaly data, where the practical locations of satellite magnetic anomaly data shown in Section 4 are utilized as synthetic observation points.

3.3 Experimental Results and Analysis

3.3.1 Modeling effect analysis

To test the correspondence between our derived ES model and the original reference SH model SH185 (i.e., to assess how well the ES model could reproduce the maximum SH degree), we performed a difference analysis and selected two extremes: magnetic anomalies ΔT at altitudes of 20 and 500 km (relative to the Earth’s mean radius of 6371.2 km). The choice of 20 km instead of 0 km was because forward calculations by tesserooids below 20 km are very slow, which causes the computational cost to become prohibitive. We compared the forward results of the ES model with those of the SH185 model, from degree 16 to 185, and calculated the root mean square error (RMSE) of the residuals as a function of the SH degree. A smaller RMSE indicates a better fit of the model. As shown in Figure S5, the results showed that at an altitude of 20 km, the RMSE was minimized at degree 72, indicating that the ES model could accurately reproduce the SH signals from degree 16 to 72. Although the RMSE was not zero, the reproduction was considered sufficiently accurate. At an altitude of 500 km, the RMSE approached zero after degree 60 because long-wavelength components dominate at high altitudes, with few or no short-wavelength signals. Therefore, the ES model could reproduce signals from degree 16 to 60 or even higher at 500 km altitude. Overall, we concluded that the ES model could reproduce the signal components corresponding to degrees 16–72 of the SH model at altitudes ranging from 20 to 500 km. It should be noted that the recovery capability of the resolution and the accuracy mainly depend on the quality (e.g., the spatial distribution and the errors) of the satellite magnetic anomaly data. The results indicated that the highest resolution that could reliably be recovered by CSES-1 and MSS-1 magnetic anomaly data might be near the maximum SH degree of 72.

At the same time, to test the spatial-domain representation accuracy of our modeling method, we performed forward modeling of magnetic anomalies ΔT at altitudes of 500, 300, 100, 50, and 20 km, as shown in Figure 1. The magnetic anomalies ΔT here are the projections of the three components of the vector onto the direction of the main magnetic field. The magnetic anomalies ΔT from degrees 16 to 72 of reference model SH185 were forward calculated for the comparison. For the ES model, we used the forward modeling method based on magnetic dipoles for altitudes above 300 km and the adaptive Gauss–Legendre integration tech-

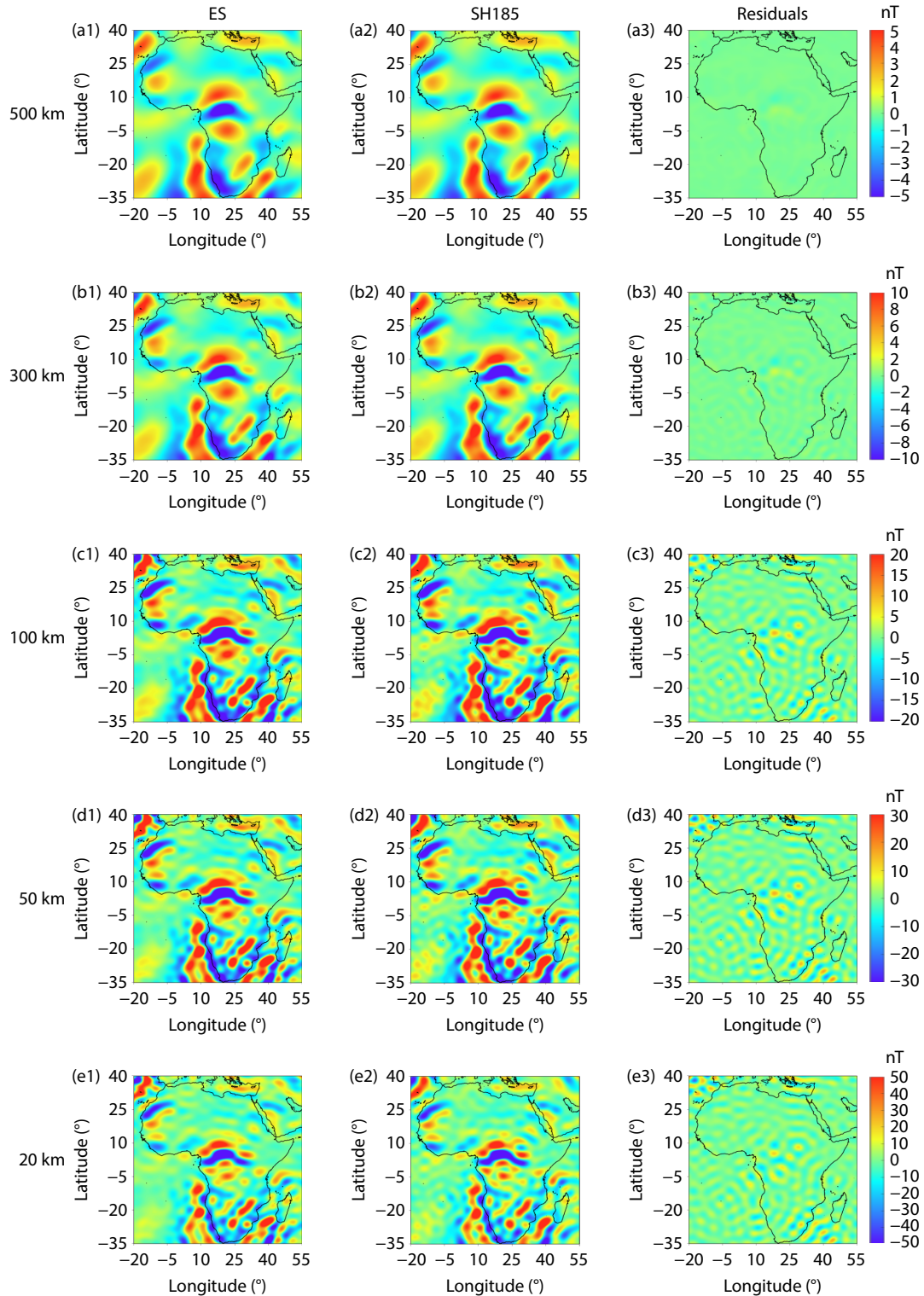


Figure 1. Forward modeling magnetic anomalies from the ES and SH185 models are shown at different altitudes: (a) 500 km, (b) 300 km, (c) 100 km, (d) 50 km, and (e) 20 km.

nique for altitudes below 300 km. The results indicated that, at altitudes ranging from 20 to 500 km, the ES model was generally consistent with the SH185 model for signals of degrees 16–72. The area with the largest difference was the Bangui magnetic anomaly area, which appeared at an altitude of 500 km and was more obvious in the downward continuation. The distribution and

morphology of the residuals between the two models suggested that the residuals primarily arose from oscillatory phenomena caused by the truncation of the SH model. In contrast, in the ES model, because of the use of the flattest objective function constraint, the oscillation phenomenon was reduced or eliminated.

3.3.2 Test of the RTP

To test the validity and accuracy of the RTP for magnetic anomalies by the ES method proposed in this study, we set the inclination of the ES to be vertically downward and conducted forward modeling of magnetic anomalies at 500, 300, 100, 50, and 20 km. Additionally, we applied the RTP to the simulated lithosphere from Section

3.1, using signal components of degrees 16–72 as a control group to validate the RTP effect. The results of the RTP magnetic anomaly fields by our ES method, the theoretical RTP fields, and the prediction residuals are presented in Figure 2. The results indicated that the RTP effect in this study was consistent with the control group on a large scale; however, the residuals were rela-

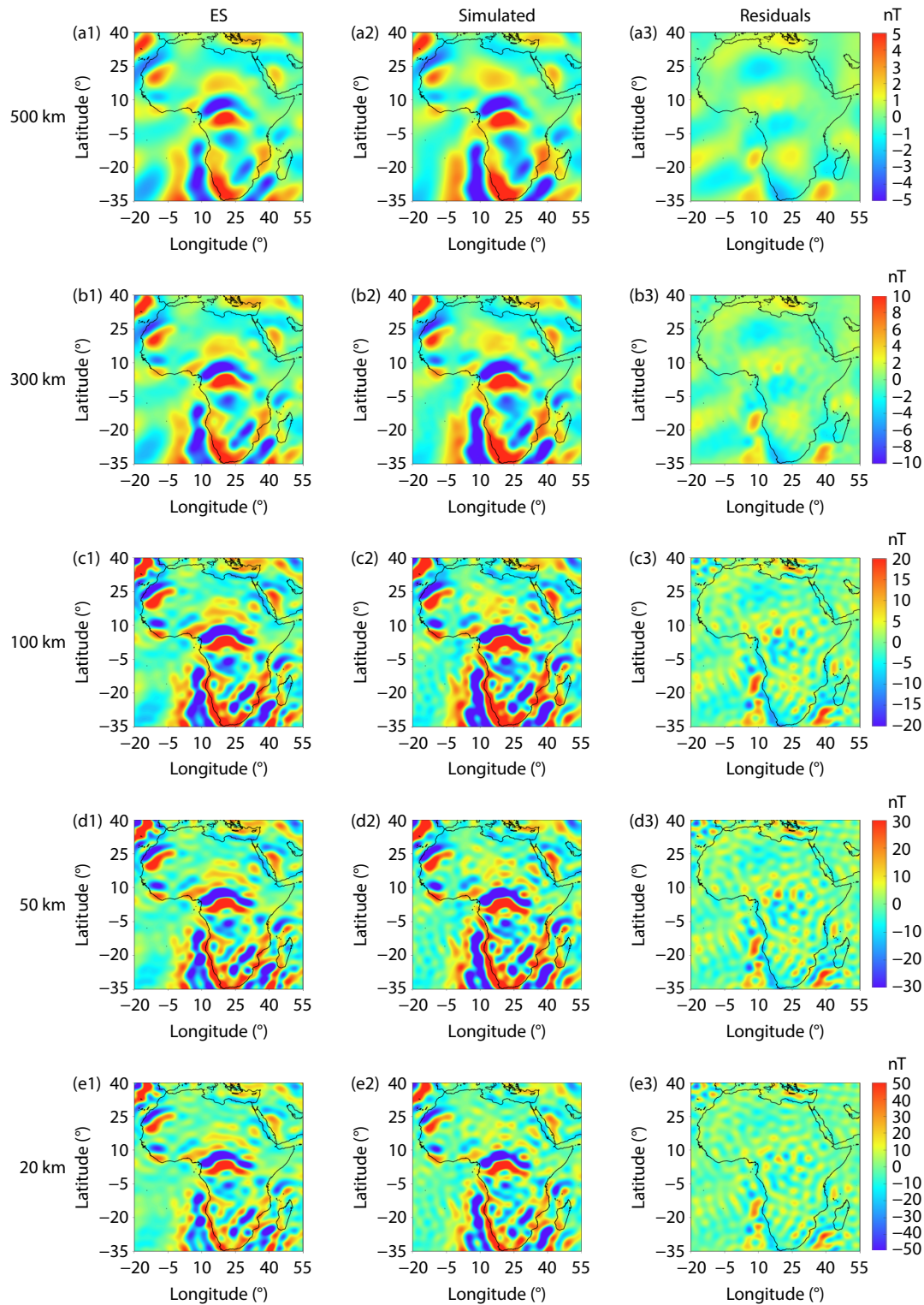


Figure 2. Forward modeling RTP magnetic anomalies from the ES model and the simulated magnetic lithosphere model are presented at different altitudes: (a) 500 km, (b) 300 km, (c) 100 km, (d) 50 km, and (e) 20 km.

tively large. This issue mainly arose for two reasons. One was the inconsistency of the spatial resolution between the forward calculation results by the ES model and by the SH model. The other was the truncation of the SH functions, which led to oscillatory phenomena.

3.3.3 Data fitting analysis

Next, we analyzed the data fitting results. The Gaussian noise introduced during the simulation experiment is shown on the left side of Figure S6, whereas the right side illustrates the residuals between the original satellite magnetic anomaly data used for inversion and the data reconstructed from the forward modeling of the ES model obtained. The frequency distribution of the residuals and some statistical parameters are presented in Figure S7. First, Figure S6 shows that both vector and scalar data fit the synthetic observation data well. The added noise and the residuals exhibited a consistent overall trend, which proved that no overfitting or underfitting occurred in our test. Furthermore, from the statistical results in Figure S7, the distribution of the original and reconstructed data showed a similar shape, suggesting that the anomaly magnitudes and distribution patterns of the original and reconstructed data were close. The correlation coefficient between the two datasets exceeded 0.9743. This result corresponds to the narrow distribution of the frequency histogram of the reconstruction residuals, which was almost entirely concentrated around 0 nT. The RMSE of the fitting residuals was less than ± 0.37 nT, and the mean residual value was 0 nT, consistent with the mean value of 0 nT of the Gaussian noise. This result demonstrated that our ES model achieved a good fit with the synthetic data.

3.3.4 Impact of the length scales

The discussion on length scales is crucial for determining the final model. To suppress the Backus effect and the instability of inversion in low magnetic latitude regions, it is essential to select a reasonable value — neither too small, which would result in inadequate suppression, nor too large, which would overly smooth the model and detach it from reality. Through testing, we selected three length scale values, $10^{4.5}$, 10^5 , and $10^{5.5}$, and conducted inversions accordingly. Because of the attenuation property of the potential field, the details of the magnetic field closer to the ES became more pronounced; thus, we compared the forward modeling magnetic anomaly ΔT at an altitude of 20 km by using the Gauss–Legendre integration technique. As a theoretical control

group, we also forward calculated the magnetic anomaly field at the same altitude by using the SH185 model from degrees 16 to 72.

As shown in Figure 3, for the simulated experimental modeling, the selection of 10^5 proved to be more suitable, aligning well with the theoretical control group while effectively suppressing oscillatory phenomena. Conversely, the results using $10^{4.5}$ contained more details but may not be realistic. Areas of particular concern were $[35^\circ\text{S}–5^\circ\text{S}, 20^\circ\text{W}–5^\circ\text{W}]$, where larger patches appeared, unlike true scalar data. The results for $10^{5.5}$ were smoother and showed significant discrepancies from the control group. In particular, the Bangui magnetic anomaly was too smooth and inconsistent with the actual results. This was mainly due to the influence of the flattest model function, which reduced many details. Therefore, the specific length scales need to be tested to find an optimal value that can effectively suppress the Backus effect and instability in low magnetic latitude regions without deviating from the actual situation.

3.3.5 Experimental summary

The ES modeling method proposed in this study was tested through synthetic data based on the simulated magnetic lithosphere model. The results demonstrated that the method could effectively fit observational data, accurately representing the magnetic anomaly signals produced by the simulated magnetic lithosphere model in the altitude range of 20 to 500 km for SH degrees 16–72. Additionally, the method effectively suppressed oscillatory phenomena in the downward continuation by forward modeling. Furthermore, the method allowed the RTP magnetic anomaly field to be obtained, providing a valuable reference for geophysical and geological interpretation. Most important, the selection of the length scale required specific testing to optimally balance the minimum model and the flattest model.

4. Satellite Magnetic Anomaly Data

In this study, we utilized geomagnetic measurements from the CSES-1 and MSS-1 missions. The CSES-1, launched in February 2018, is equipped with high-precision scalar and vector magnetometers (e.g., Wang J et al., 2025) and operates in a near-polar orbit with an inclination of 97.4° at an average altitude of approximately 500 km. The MSS-1, launched in May 2023, is a collaborative effort between the Macau Special Administrative Region Government and the China National Space Administration (Livermore et

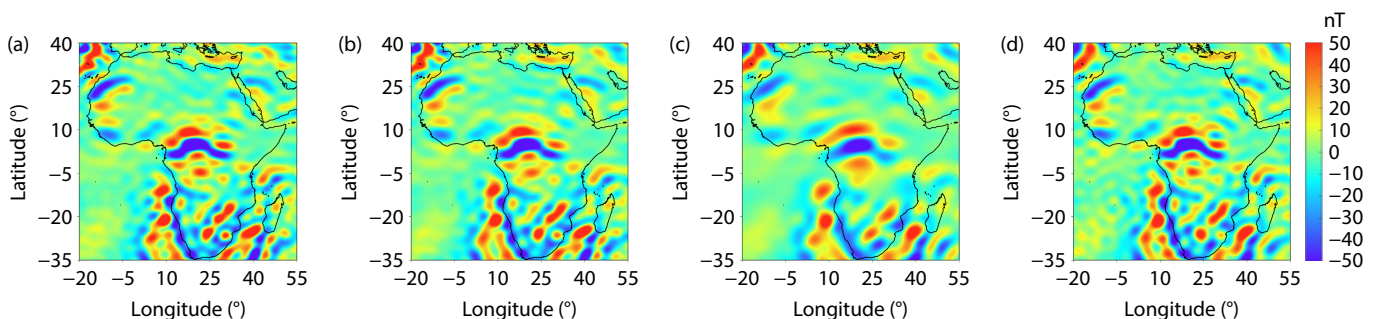


Figure 3. Forward modeling magnetic anomaly ΔT at 20 km altitude when using different length scales. In (a), (b) and (c), the length scales are $10^{4.5}$, 10^5 , and $10^{5.5}$, respectively. Panel (d) is calculated from 16 to 72 degrees of the SH185 model.

al., 2024). Although its primary objectives include marine and atmospheric observations, environmental monitoring, and scientific research, MSS-1 also carries high-precision magnetometers, with an orbital inclination of 41° and an average altitude of approximately 450 km. The MSS-1 mission plays a critical role in advancing highly accurate models of the Earth's magnetic field (e.g., Zhang K, 2023).

We selected scalar data from CSES-1 covering January to December 2019 and both scalar and vector data from MSS-1 collected between November 2023 and April 2024. Using these satellite magnetic measurement data, we obtained lithospheric magnetic anomaly signals of SH degree 16 and above after removing the main field components of degree 15 and below, as well as external field components such as ionospheric and magnetospheric contributions, and performing data selection and filtering. These satellite magnetic anomaly data could then be used for large-scale lithospheric magnetic field modeling.

Data selection adhered to standards comparable to those used in the construction of the CHAOS-7 model (Finlay et al., 2020). Specifically, CSES-1 data were sampled every 30 seconds, whereas MSS-1 data were sampled every 10 seconds. Additional selection criteria included (i) measurements taken only in dark regions with the Sun at least 10° below the horizon; and (ii) K_p and RC index with variations limited to 2 nT/hour or less. To preprocess the selected data, we removed the main field and external field components by using the CHAOS-7 model. Subsequently, we subtracted the reliable low-degree components (degrees 16–80) of the LCS-1 model and applied a high-pass filter up to SH degree 15 to eliminate signals of SH degree 15 and below. The reliable low-degree components of degrees 16–80 were then reintroduced to obtain the lithospheric magnetic anomaly signals. Despite this process, many interfering components remained in the processed lithospheric magnetic anomaly signal data. For scalar data, no further selection was conducted. However, for vector data, additional selection was necessary based on the following criteria: (i) Vector field data were used only equatorward of $\pm 55^\circ$ quasi-dipole (QD) latitude; (ii) interplanetary magnetic field (IMF) B_z at the magnetopause averaged over the previous 2 hours was positive; (iii) IMF B_y at the magnetopause averaged over the previous 2 hours was less than +3 nT when the QD latitude was positive (northern QD hemisphere, i.e., $-\infty < B_y < 3$ nT), whereas when the QD latitude was negative (southern QD hemisphere), it was greater than -3 nT (i.e., $-3 \text{ nT} < B_y < \infty$; e.g., Friis-Christensen et al., 2017); (iv) the merging of the electric field at the magnetopause was averaged over the previous 2 hours, $E_m \leq 0.8$ mV/m; (v) the threshold for vector data was set at ± 50 nT.

Figure 4 illustrates the standard deviations of the residuals between the selected data and the LCS-1 model (Olsen et al., 2017), binned in 2.5° intervals of QD latitude. To evaluate the data quality, these standard deviations were calculated, and the standard deviation was fitted by using a 10th-degree polynomial. Because the focus of this modeling effort was the African region, data selection was further restricted to a geographic area spanning 55°W – 90°E in longitude and 70°S – 75°N in latitude. This selection yielded 205,549 scalar data points (85,355 from CSES-1 and 120,194 from MSS-1) and 118,430 vector triplets (all from MSS-1).

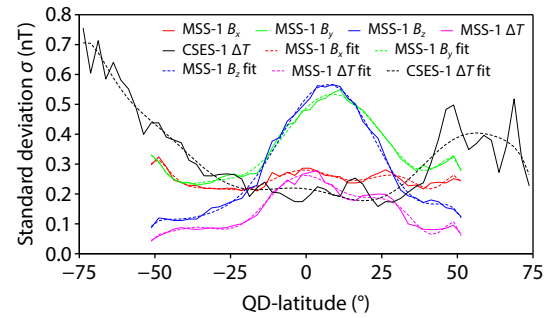


Figure 4. The standard deviations of the CSES-1 and MSS-1 residuals. The solid line is the true standard deviation, and the dotted line is the fitted standard deviation.

5. Regional Lithospheric Magnetic Field Model over Africa

At first, we analyzed the data fitting between our derived model and the actual data. Figure S8 displays the residuals between the observed data and the forward modeled data from the LCS-1 model and from our derived ES model. Figure S9 presents the histograms and key statistical parameters for the vector and scalar data utilized in the model. The statistical analysis revealed that the range of extreme values for both the original and reconstructed data were within ± 10 nT, featuring a narrow distribution of data magnitudes. This result was consistent with the expected pattern of lithospheric magnetic anomalies decreasing with distance. The correlation coefficient between the original and the reconstructed data (the forward modeling data of the ES model) was high (>0.9632), and the residuals were mostly centered on 0 nT. The RMSE of the residuals was less than ± 0.37 nT. However, the residuals still contained some long-wavelength components that did not physically couple with the data. Vervelidou et al. (2018) suggested these residuals may be due to unremoved ionospheric and magnetospheric signals in the data processing. Overall, the ES model demonstrated good data fit, with small reconstruction errors.

The ES model can provide multiple-component predictions of the lithospheric magnetic field at different altitudes. Figure 5 displays the forward predicted vector data from the ES model at altitudes of 500, 300, 100, 50, and 20 km. It is evident that the high-frequency components of the magnetic anomalies decay more rapidly with increasing altitudes, whereas the lithospheric magnetic field becomes progressively smoother and more subdued as the altitude increases, with decreasing magnitude. This observation aligns with the physical nature of the spatial variations in the potential field.

The RTP is a process that transforms total magnetic anomalies from oblique magnetization to vertical magnetization, which is helpful for geophysical and geological interpretation. We conducted forward modeling of the magnetic anomaly ΔT , setting the magnetic inclination angle to 90° to obtain the RTP magnetic anomaly ΔT_{RTP} , as illustrated in Figure 6. The results indicated that compared with those before the RTP, the magnetic anomalies after the RTP more accurately reflected the boundary range and morphological characteristics of the magnetic sources, providing a valuable reference for subsequent geological interpretation.

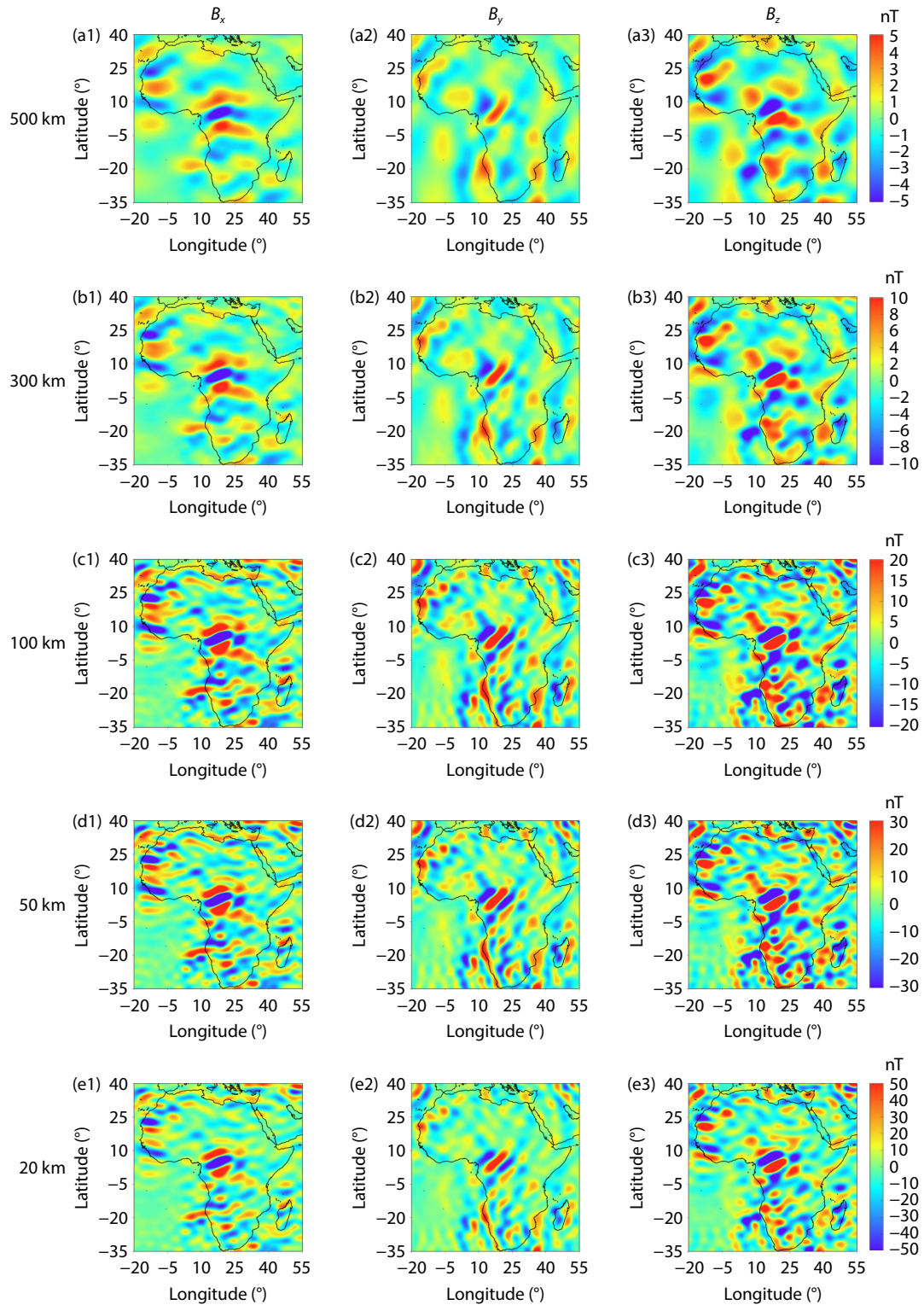


Figure 5. Vector distribution of the lithospheric magnetic field predicted by the ES model at different altitudes: (a) 500 km, (b) 300 km, (c) 100 km, (d) 50 km, and (e) 20 km.

6. Discussion

6.1 Influence of the Length Scales

The length scales primarily affected the flatness of the ES model and the magnetic fields. Similar to the simulation experiments, we selected three length scale values, $10^{4.5}$, 10^5 , and $10^{5.5}$, and

conducted the respective inversions. We also compared the forward modeling magnetic anomaly ΔT at an altitude of 20 km. As with the synthetic data test, we compared the magnetic anomalies by using the LCS-1 model. As shown in Figure 7, for modeling from the measured data, the selection of 10^5 proved to be more suitable, with its details aligning more closely with the

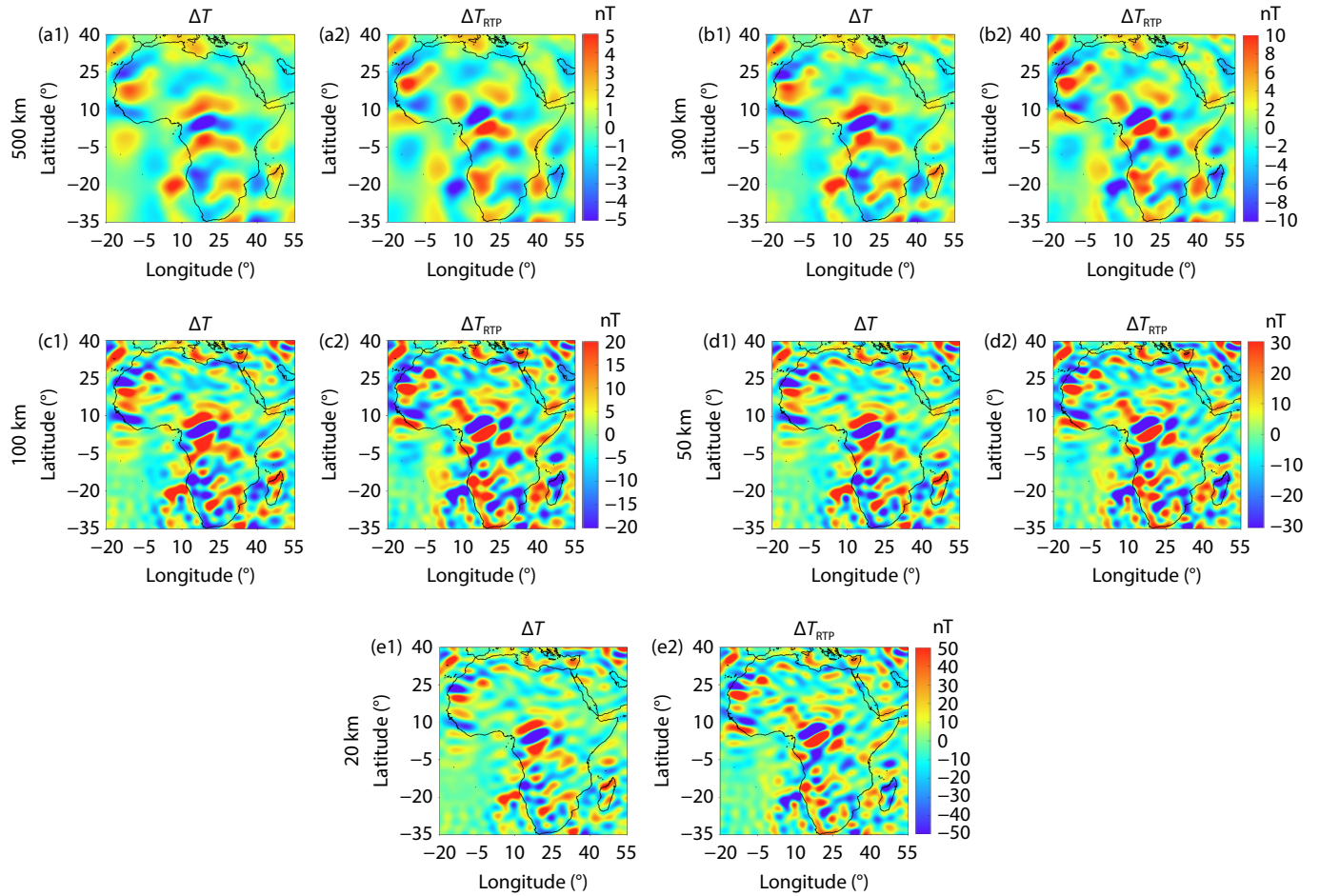


Figure 6. The ΔT magnetic anomaly fields before and after the RTP at different altitudes: (a) 500 km, (b) 300 km, (c) 100 km, (d) 50 km, and (e) 20 km.

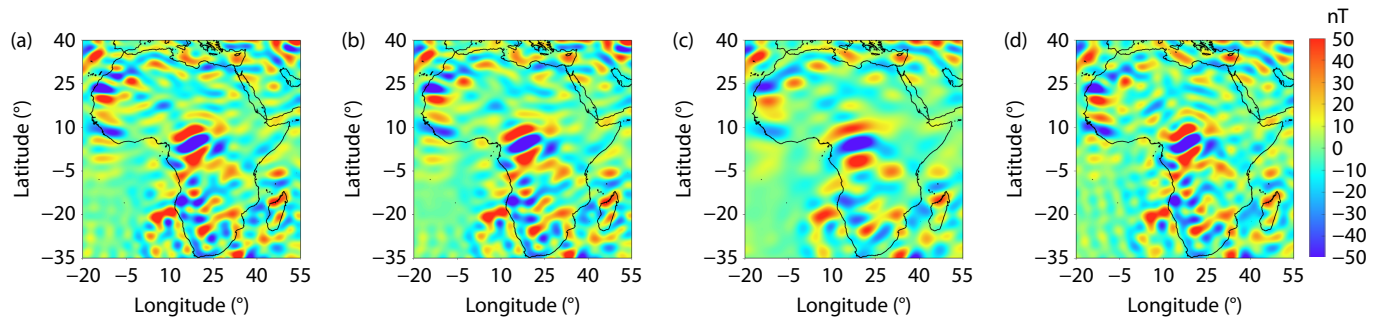


Figure 7. Predicted ΔT magnetic anomaly fields by the ES model with different length scales and by the LCS-1 model. Length scale values of (a) $10^{4.5}$, (b) 10^5 , and (c) $10^{5.5}$, respectively. (d) Calculated from 16–72 degrees of the LCS-1 model.

control group, the LCS-1 model. After careful inspection, we concentrated on the area $[35^{\circ}\text{S}–5^{\circ}\text{S}, 20^{\circ}\text{W}–5^{\circ}\text{W}]$ and found that using 10^5 reduced the oscillation phenomenon but did not become too smooth, so it was a more suitable choice. In contrast, the results by $10^{5.5}$ were smoother, such as the Bangui magnetic anomaly. This reduced important details that we believe represented the actual situation.

6.2 Comparison with Other Models

In this section, we compare our derived ES model with the LCS-1 (Olsen et al., 2017), MF7 (Maus, 2010), and CM6 (Sabaka et al.,

2020) models. For the ES model, forward modeling at altitudes above 300 km utilized magnetic dipoles, whereas tesseraoids with Gaussian–Legendre integration techniques were used for calculations below 300 km. We performed forward modeling of ΔT magnetic anomalies at altitudes of 500, 300, 100, 50, and 20 km, as shown in Figure 8. For the SH models LCS-1 (maximum degree 185), MF7 (maximum degree 133), and CM6 (maximum degree 120), forward modeling was performed for degrees 16–72. At altitudes above 300 km, our ES model was in good agreement with the three SH models. Below 300 km, our ES model generally aligned with the LCS-1, MF7, and CM6 models, with minor

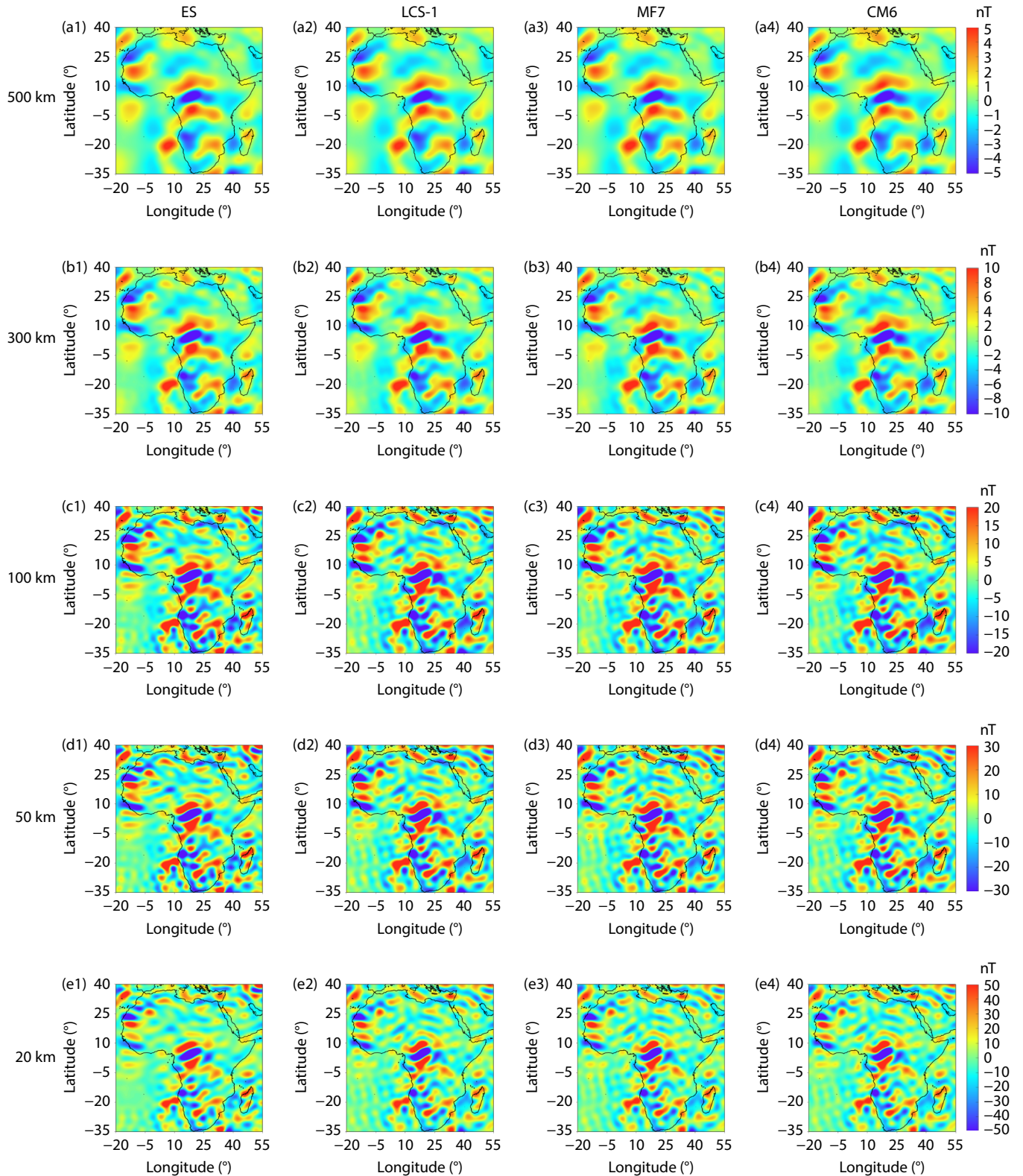


Figure 8. The ΔT magnetic anomaly fields at different altitudes calculated by the ES, LCS-1, MF7, and CM6 models: (a) 500 km, (b) 300 km, (c) 100 km, (d) 50 km, and (e) 20 km.

discrepancies observed in the Bangui region, which were considered acceptable.

In addition, to compare the downward continuation of the four models, we focused on the region [35°S–5°S, 20°W–5°W]. The

downward continuation results of the ES model at altitudes below 100 km were smoother, with no obvious oscillation phenomenon, in contrast to the maps produced by the SH models. We concluded based on this outcome that as for the downward continuation at altitudes below 100 km, the ES model resulted in

smoother reconstructed magnetic anomaly maps with fewer false anomalies and oscillations.

6.3 Comparison with the Geological Map

The ES method offered the advantage of deriving the lateral magnetization contrast distribution. In this study, we used magnetic dipoles, which were converted into magnetization. By multiplying these values by the crustal thickness, we obtained the distribution of radially integrated susceptibility anomalies (Figure 9a). We could see that, compared with the geological map of Africa in Figure 9b, positive susceptibility anomalies were typically associated with the Archean and Proterozoic eras, whereas negative anomalies are generally linked to the Paleozoic, Mesozoic, and Cenozoic eras. The positive susceptibility anomalies in the Archean and Proterozoic regions likely reflect the abundance of ferromagnetic minerals in the lithosphere and a relatively thick and cold magnetic lithosphere, which suggests the relatively stable tectonic setting. In contrast, negative susceptibility anomalies during later periods may have resulted from the relatively hot and thinned magnetic lithosphere attributable to the latest tectonic activity and thermal history. A detailed geological interpretation is beyond the scope of the study, but in short, the susceptibility contrast distribution correlates well with the large-scale tectonic province framework. This result provides independent evidence proving the reliability of our derived ES model and the validity of the proposed ES method.

7. Conclusions and Outlook

This study focused on the ES method in spherical coordinates for modeling the lithospheric magnetic field, integrating an alternative regularization term including the minimum and flattest models to balance the data misfit and complexity of the model. The method was initially validated by using synthetic data and then applied to MSS-1 and CSES-1 magnetic anomaly data for modeling the large-scale lithospheric magnetic field in Africa and surroundings.

The synthetic data tests confirmed the reliability of the proposed method, demonstrating its effectiveness in fitting the data. With the selection of appropriate length scales, the method successfully mitigated the Backus effect and reduced inversion instability in low magnetic latitudes. Using actual satellite magnetic anomaly data from MSS-1 and CSES-1 magnetic measurements, we constructed a robust ES model for the regional lithospheric magnetic field with large scales over the African region. On the basis of the ES model, the multiple-component grids of the magnetic anomaly field at altitudes from 20 to 500 km could be predicted via forward calculation. In particular, the RTP magnetic anomaly field could be calculated by setting radial polarizations. Compared with other global SH models in the literature, our ES model showed strong agreement and superior stability at altitudes below 100 km. Moreover, by means of the ES method, lateral variations in the apparent susceptibility contrast could be obtained. The recovered susceptibility contrast distribution illustrates good correlation with regional tectonic provinces in Africa and surroundings.

In the next phase of work, we will integrate more data from the CHAMP and *Swarm* satellites, as well as spatial differential gradient data and near-surface magnetic anomaly data, to develop a higher resolution regional lithospheric magnetic field model. Additionally, the upcoming MSS-2 (Jiang Y et al., 2023) and CSES-2 (Wang J et al., 2024b) — especially the MSS-2, with its two satellites in two highly elliptical orbits of a 90° inclination with a perigee of approximately 200 km and an apogee of 1500 km (Livermore et al., 2024) — will provide an unprecedented opportunity to enhance the quality of lithospheric magnetic field modeling (e.g., Jiang Y et al., 2023; Olsen, 2023).

Supplementary Material

Additional Supplementary Material can be found in the online version of this article.

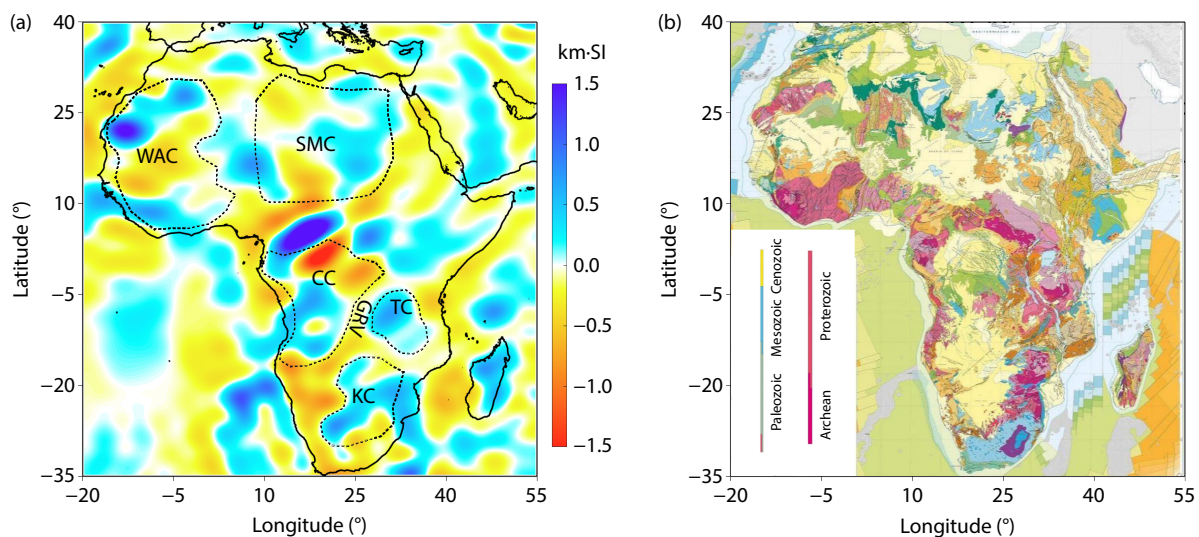


Figure 9. (a) The radially integrated susceptibility anomaly in Africa (WAC: West African Craton; CC: Congo Craton; KC: Kalahari Craton; TC: Tanzania Craton; SMC: Saharan Metacraton; GRV: Great Rift Valley; the black lines represent the outline of the Craton boundary); (b) geological map of Africa (1:1 million, © 2016, BRGM/French Geological Survey, <https://www.brgm.fr/en/reference-completed-project/new-edition-110000000-geological-map-africa>).

Acknowledgments

We would like to thank anonymous reviewers and the editor for their valuable suggestion and helpful comments that have improved the clarity and standardability of the manuscript. This work was supported by the National Natural Science Foundation of China (Grant Nos. 42250103 and 42174090), the Opening Fund of Key Laboratory of Geological Survey and Evaluation of Ministry of Education (Grant No. GLAB2023ZR02), and the MOST Special Fund from the State Key Laboratory of Geological Processes and Mineral Resources (Grant No. MSFGPMR2022-4). We also thank the CSES-1 (<https://leos.ac.cn>) and MSS-1 (<https://mss.must.edu.mo>) missions for providing satellite magnetic measurement data.

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